# Stochastic Digital Twin of a Composite Plate for Predicting Lamb Wave Propagation

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## Abstract

This work presents a framework for stochastic updating for verifying and validating a finite element (FE)-based model - the Digital-Twin - of a composite plate, considering temperature influence on Lamb wave propagation. It begins with a deterministic updating procedure to find optimal mechanical properties, followed by a stochastic updating procedure to obtain probability density functions for meaningful parameters. The stochastic updating procedure is divided into two steps: a sensitivity analysis using Sobol Indices and a Bayesian inference process using Markov-chain Monte Carlo (MCMC) simulations and the Metropolis-Hastings sampling algorithm. To reduce the computational time required for the MCMC process, the work proposes using a surrogate model based on artificial neural networks (ANNs). The ANN can be trained using parallelized Monte Carlo simulations, in contrast to the sequential nature of the MCMC process. This approach reduced the time required for updating rounds by 450 times in the studied case without compromising the accuracy of the resulting probability density functions for model parameters.

## 1 Introduction & Methodology Framework

Lamb waves, which are ultrasonic guided elastic waves, play a vital role in structural health monitoring and evaluation of plate-like structures. However, accurately modeling Lamb wave propagation in composite structures is challenging due to their inherent heterogeneity and the influence of varying physical properties affected by the environment. To address these challenges and ensure consistency with experimental data, numerical models must be updated using stochastic approaches that account for environmental variations.

In this paper, we propose a Bayesian framework for stochastically updating a numerical model for Lamb wave propagation in composite structures, forming a Stochastic Digital Twin. The framework, as illustrated in Fig. 1, involves a modified least-squares method to identify mechanical properties and a sensitivity analysis using Sobol indices to assess the influence of identified parameters on the model response. Parameters with low contributions are considered determined quantities, while those with high contributions are updated using Bayesian inference with non-informative prior distributions derived from the deterministic optimization scheme. The Bayesian inference process is performed using Markov-Chain Monte Carlo (MCMC) simulations and the Metropolis-Hastings sampling algorithm. To overcome computational limitations, we propose an artificial neural network (ANN) surrogate model that can be trained using parallelized Monte Carlo simulations. This approach significantly reduces the time required for updating the model without compromising the accuracy of resulting probability density functions for model parameters.

By combining Bayesian inference, stochastic updating, and a digital twin framework, our approach enables accurate prediction of Lamb wave propagation in composite plates, considering uncertainties and variations introduced by the environment. The resulting Stochastic Digital Twin holds promise for enhanced structural health monitoring and evaluation of composite structures.

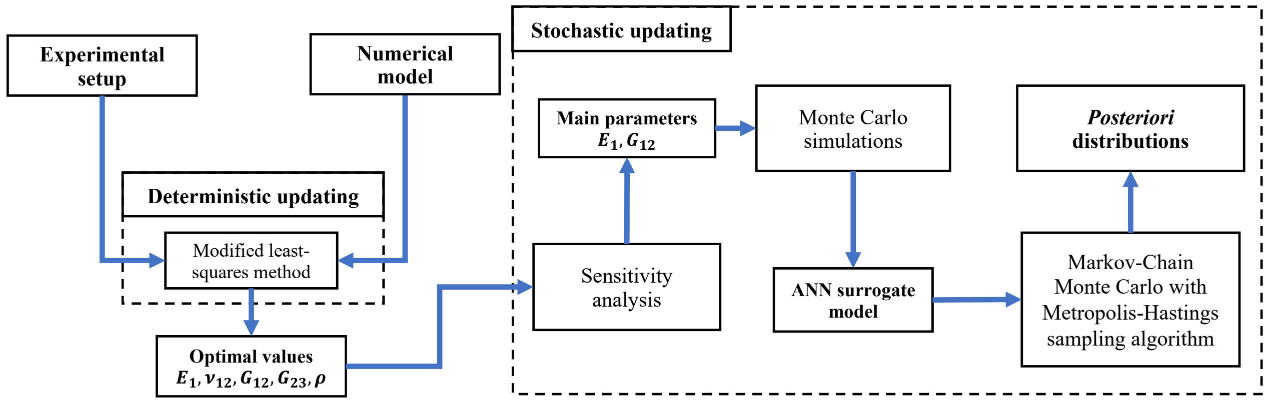



Figure 1: Proposed framework structure with deterministic and stochastic updating schemes.

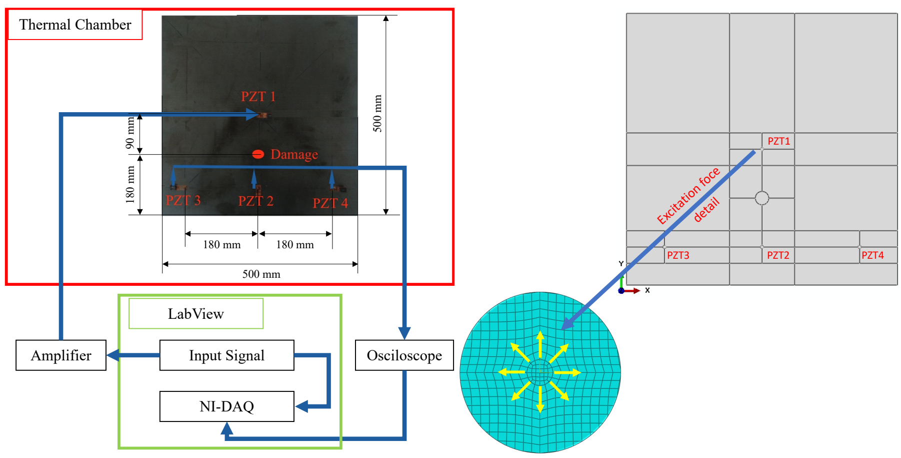
## 2 Experimental Setup and Numerical Model

The experimental setup shown in Fig. 2a consists of a 500 x 500 x 2 mm$^3$ carbon fiber reinforced polymer (CFRP) plate with 10 layers of plain weave fibers. Four 6.35 mm PbZrTi (PZT) SMART Layers are bonded to the plate with epoxy resin. PZT 1 is the actuator, whereas PZTs 2, 3, and 4 serve as sensors. Additional information on this dataset is available on GitHub [1].

The digital twin of the CFRP plate is created using a finite element (FE) model with continuum shell elements (SCR8) in ABAQUS/Explicit, as illustrated by Fig. 2b. The model represents the CFRP plate with ply-based properties and an orthotropic laminate using 3 integration points per lamina. A spatial resolution of at least 20 nodes per wavelength and a time step based on the maximum expected frequency (250 kHz) are used for numerical stability. Strain data within the sensor region is transformed to voltage using the method proposed by Sirohi and Chopra [2].

## 3 Application of the Identification Framework for Digital Twin Development

By combining experimental data with numerical modeling techniques, the framework enables the development of a digital twin that closely replicates the behavior of the physical structure. In the initial deterministic updating process, the mechanical properties of the materials in the numerical model are adjusted to match the response signals from sensors 2 and 3 with the experimental results, as shown in Fig. 3a(a). This step involves




(a) (b)

Figure 2: Experimental setup and numerical model.

updating parameters such as Young's modulus in two perpendicular directions ($E_1$ and $E_2$), Poisson's ratio ($\nu_{12}$), shear moduli in and out of the plane ($G_{12}$ and $G_{23}$, respectively), and density ($\rho$). These material properties are critical in accurately simulating the behavior of the structure. The initial range of values for these parameters is determined based on previous work. By refining these properties, the digital twin can better capture the structural response. Figure 3a(a) compares the model response using initial and optimized properties with the experimental results.

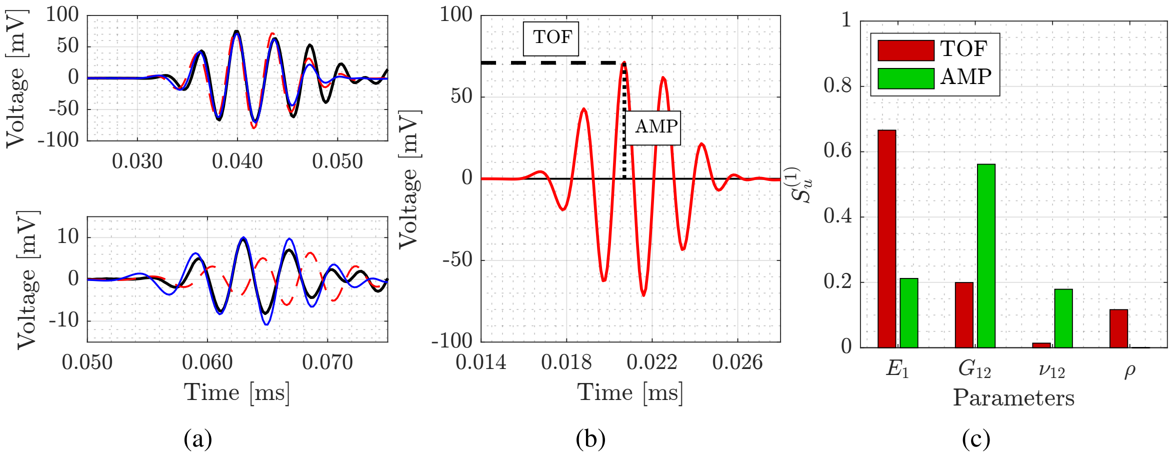



Figure 3: Deterministic updating and sensitivity analysis results. (a) comparison between initial (--) and optimal models (—) and experimental (—) results for PZTs 2 (above) and 3 (below); metrics used in sensitivity analysis; and (c) first-order Sobol indices

Following the deterministic adjustment, a sensitivity analysis is conducted to evaluate the influence of each parameter on the digital twin's behavior. Sobol indices are employed to quantify the impact of parameters such as $E_1$, $\nu_{12}$, $G_{12}$, and $\rho$ on the model's response. Perturbing each parameter within a certain range, the sensitivity analysis measures metrics such as time of flight (TOF) and maximum envelope value (AMP) of the first wave packet, as shown in Figs. 3b and 3b. The parameters $\nu$ and $\rho$ are determined based on the optimization results, whereas the remaining parameters are considered undetermined and need to be treated as random variables.

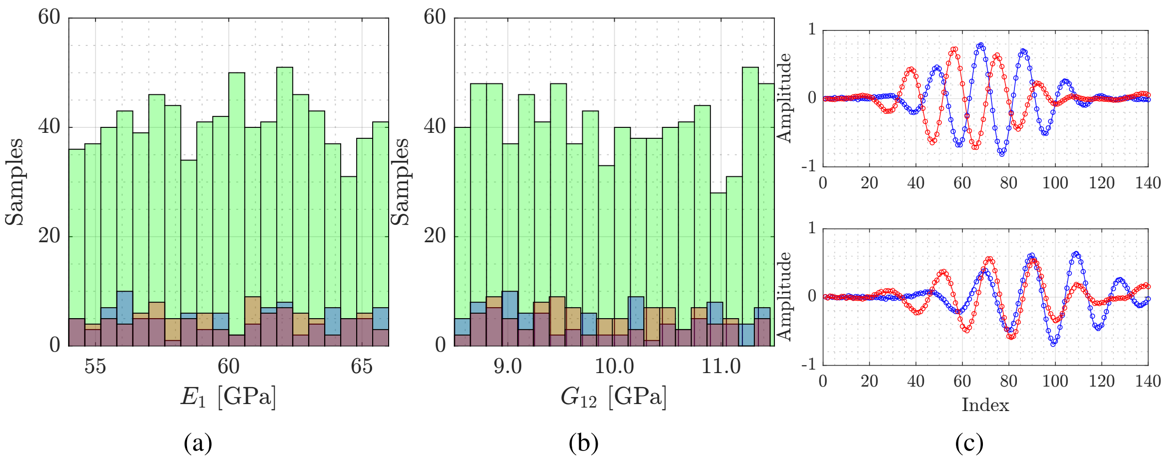
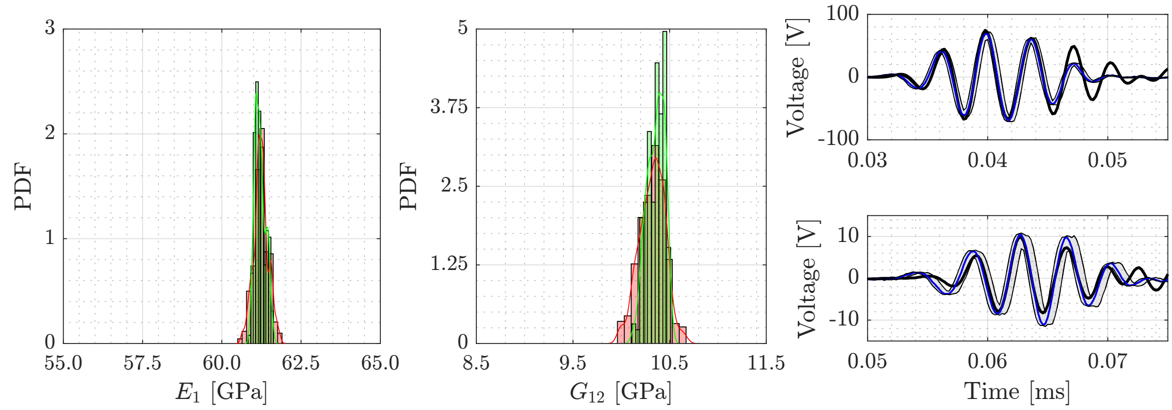



Figure 4: Surrogate model evaluation. (a) $E_1$ Histogram for training ( ■ ), validation( ■ ) and test( ■ ); (b) $G_{12}$ Histogram; and (c) Comparison between surrogate model (∘) and FE model (—) for different values of test data

To expedite the random-walking process in the Bayesian updating phase, an artificial neural network (ANN) surrogate model is introduced as a substitute for the computationally expensive finite element (FE) model. The ANN surrogate model, comprising the multilayer perceptron with two fully connected hidden layers, takes $E_1$ and $G_{12}$ as input parameters and predicts the time series output. Remarkably, the surrogate model closely replicates the numerical model's response without noticeable differences. Furthermore, the surrogate model demonstrates significantly faster evaluation times, with each prediction taking less than a second compared to several minutes for the FE model.

The Bayesian inference process is then applied to experimental data obtained at 20°C, utilizing both the numerical model and the surrogate model. By employing a random walk algorithm, the parameters in the identification framework are iteratively updated, starting from the center of the parameter search space. A burn-in period is implemented to discard the initial part of the sampling process, ensuring convergence of the algorithm. The resulting probability distributions for $E_1$ and $G_{12}$ are compared between the numerical model and the surrogate model. Both are illustrated in Figs. 5a and 5b, respectively. Surprisingly, both models yield similar distributions, validating the surrogate model's effectiveness in capturing the uncertainty of the digital twin's parameters.




(a) (b) (c)

Figure 5: Restuls for the stochastic updating procedure: (a) $E_1$ posterior histograms and PDFs using the FE model (■) and the ANN (■), (b) $G_{12}$ posterior histograms and PDFs , and (c) Experimental (—), model mean result (—) and model confidence interval ( ■ ) signal comparison for 20°C in PZT 2 and PZT 3

The confidence interval of the digital twin's output can be determined by evaluating the models with values within the 99%percentile of the results, as shown in Fig. 5(c) . This uncertainty quantification enables a better understanding of the digital twin's predictions and facilitates decision-making based on the confidence level of the model's output.

# 4 Final Remarks

In this study, we presented a Bayesian framework for stochastically updating a numerical model for Lamb wave propagation in composite structures, emphasizing the aspect of digital twin development. By combining a modified least-squares method, sensitivity analysis using Sobol indices, and Bayesian inference, accurate probability density functions for model parameters were obtained. The use of MCMC simulations and the Metropolis-Hastings sampling algorithm ensured robust parameter estimation. Additionally, the introduction of an ANN surrogate model enabled faster computations through parallelized Monte Carlo simulations, without compromising the accuracy of the resulting probability density functions. This methodology provides valuable insights for adjusting Lamb wave signals in composite structures.

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