Noise robust gearbox defect diagnosis

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Abstract

Vibratory analysis focused on the detection of periodically impulsive signatures caused by tooth crack of a gearbox in early stage is of a great interest in industrial applications. This problem has received great attention in the last decades. A wide variety of model-based approaches requiring advanced methods for processing and analyzing the vibration signal for early fault detection have been proposed. For the vibration signal processing about gearbox fault diagnosis, the noise caused by data acquisition systems and other machine elements, must be properly filtered from the recorded vibration signal. A novel gearbox fault diagnosis method taking into account modeling errors of vibration signal, additive noises and the knowledge of the default frequency is proposed.

A new model of faulty vibration signal from those usually used for gearbox fault diagnosis is introduced. Then an optimal estimator, combining simultaneously an optimal strategy for noise filtering of the recorded signal and the estimation of fault signal by a deconvolution approach based on the least squares optimization, is determined.

The new approach was applied to simulated faulty vibration signal under constant speed condition and the results showed the robustness of the proposed technique against to noise and its reliability for fault diagnosis.

1 Introduction

Gears are the main elements of power transmission used in many industrial applications and are subjected to significant stresses and loading. it is also a main element where faults often occur in the operating conditions of many types of machinery, such as wind turbines for instance [1]. A gearbox failure can cause system failures so that their health is critical to the proper operation of the entire mechanical system.

Most techniques for gear fault diagnosis are based on the analysis of vibration signals recorded from the gearbox. These signals carry important information which is very useful in early detection of defects [3],[7]. The gearbox vibration signal includes tooth meshing harmonics and sideband frequencies. Depending on the bandwidth of the sidebands, there can be aliasing between the sidebands of neighbouring meshing harmonics. The recorded vibration signal usually comprises also contributions from other parts of the machine or other machines, that can be considered as noise.

For gear fault diagnosis the narrow demodulation technique is often used. It consists in applying a band-pass filter around one meshing harmonic in order to extract the demodulated signal component with the features induced by the gear crack. The quality of the estimated demodulated signal depends on the bandwidth of the filter. In some cases the extracted demodulated signals can provide sufficient gear fault information for diagnosis [7]. The selection of the filter bandwidth is a key factor that influences the effectiveness of the narrow demodulation technique for gear fault detection and diagnosis. If too narrow, it misses some important information about early damage, that can appear on the high order harmonics of the modulation, but if greater it can take into account sidebands from the neighbouring meshing harmonics. Time-frequency analysis methods can provide both time and frequency information of signal [3]. Particularly the Short Time Fourier Transform (STFT) [10], Wavelet Transform (WT) [11] and Wigner-Ville distribution (WVD) [5] are often used in gearbox diagnosis. For non-stationary signals, the cyclostationary framework has been introduced for early gear damage detection [2]. But none of these techniques can overcome the problem induced by the aliasing between sidebands.

Motivated by earlier research for detecting and localizing cracks in gears by a signal processing approach and in order to overcome the sidebands aliasing as well as the noise problems, we previously proposed to employ H_∞ filter with linear time-varying model in order to estimate the modulation signal characterizing the failure [9]. In this work, we extend our previous work with a new approach for gear tooth crack detection technique, that allows taking into account the fact that the number of harmonics of the sidebands is unknown. The denoised vibration signal is estimated by using simultaneously an optimal filtering approach combined with a deconvolution technique [8].

The structure of the paper is as follows: Section 2 presents the methodology of the matrix formulation of gear vibration signal then the denoising and estimation technique based on optimal filtering and deconvolution approaches. In Section 3, a setup of synthetic gear vibration signals to illustrate the behavior of the proposed algorithm is explained. Section 4 presents the analysis results of the simulations and describes how a fault is identified. In Section 5 a conclusion of this paper is given.

2 The methodology

2.1 The matrix formulation of gear vibration signal

In this paper, we consider the operational defect of a gearbox (tooth wear, case wear, tooth cracks). A healthy gear in normal condition working smoothly and periodically generates a linear and periodic vibration signal [3]. The carrier signal of the gear meshing vibration, s_h, of a fault-free normal pair of gears meshing under a constant load speed can be formulated as in [7] by :

$$s_h(t) = \sum_{k=1}^{K} s_k . cos(2\pi k f_{Mesh} t + \xi_k) + v(t) \quad (1)$$

where s_k and ξ_k are the amplitude and phase of the k-*th* meshing frequency harmonic and f_{Mesh} is the meshing frequency. K is the total number of f_{Mesh} harmonics in the vibration signal and the random part of the vibration signal, $v(t)$ is supposed to be a stationary Gaussian white noise with variance σ_v^2. For example, a pinion wheelwith P teeth and rotating at frequency f_p, has a meshing frequency $f_{Mesh}=P.f_p$. The same meshing frequency $f_{Mesh}=G.f_G$ can be obtained considering the gear wheel with G teeth and rotating at frequency f_G .

For a gear localized fault, such as a fatigue crack at the root of one tooth or a broken tooth of the pinion, the occurring of an abnormal movement alters the vibration signal. In a such case, the vibration contains amplitude and phase modulation of the meshing signal. Its frequency spectrum includes sidebands, frequency components on two sides of the meshing frequency and its harmonics. The vibration signal expression is thus given as in [7] by:

$$s_f(t) = \sum_{k=1}^{K} s_k \big(1 + \beta_k(t)\big). cos(2\pi k f_{Mesh} t + \xi_k + \psi_k(t)) + v(t) \quad (2)$$

where $\beta_k(t)$ and $\psi_k(t)$ are the amplitude and phase periodic modulation functions of the k-*th* harmonic resulting of the alteration produced by the fault-induced impact.
The modelling of the amplitude modulation $\beta_k(t)$ is crucial since its choice determines the quality of the faulty diagnostic as it is mentioned in [6].

It's well known that Fourier series can decompose any periodic signal or function into the sum of simple trigonometric functions, namely function sinus and cosines. As the modulation signal $\beta_k(t)$ is a periodic process with a finite power, it can be represented by a discrete Fourier series [2].

$$\beta_k(t) = \sum_{l=1}^{\infty} a_{k,l}.cos(2\pi l f_p t + \theta_{k,l})$$

In order to take into account only a finite number of harmonics, we propose that $\beta_k(t)$ is re-written as follows:

$$\beta_k(t) = \sum_{l=1}^{L} a_{k,l}.cos(2\pi l f_p t + \theta_{k,l}) + w_k(t) \quad (3)$$

where $a_{k,l}$ and $\theta_{k,l}$ are the amplitude and phase of the l-*th* pinion frequency harmonic, f_p is the meshing frequency and $w_k(t)$ is a stationary Gaussian white noise with variance σ_w^2 that encompasses the modelling error and takes into account the unmodelled harmonic components of the modulation signal. By using Eq.3 and by applying trigonometric identities, the vibration signal in Eq.2 can be rewritten as a sum of its quadrature and in-phase components.

$$s_f(t) = \sum_{k=1}^{K} \Big(1 + \sum_{l=1}^{L} a_{k,l}.\left(\cos(\theta_{k,l}).cos(2\pi l f_p t) - sin\,(\theta_{k,l}).sin(2\pi l f_p t)\right) + w_k(t)\Big).s_k.\left(\cos(\xi_k + \psi_k(t)).cos(2\pi k f_{Mesh} t) - sin(\xi_k + \psi_k(t)).sin(2\pi k f_{Mesh} t)\right) + v(t) \quad (4)$$

In order to get a linear parametrization, the vibration signal expression in Eq.4 can be written:

$$\boldsymbol{s_f(t) = \sum_{k=1}^{K} \Big(\tilde{a}_{k,l}.cos(2\pi k f_{Mesh} t) - \tilde{b}_{k,l}.sin(2\pi k f_{Mesh} t) + \sum_{l=1}^{L} \tilde{c}_{k,l}.\cos(2\pi l f_p t).cos(2\pi k f_{Mesh} t) - \tilde{d}_{k,l}.\cos(2\pi l f_p t).sin(2\pi k f_{Mesh} t) + \tilde{e}_{k,l}.sin(2\pi l f_p t).cos(2\pi k f_{Mesh} t) + \tilde{f}_{k,l}.sin(2\pi l f_p t).sin(2\pi k f_{Mesh} t) + \tilde{w}_{k,1}.cos(2\pi k f_{Mesh} t) - \tilde{w}_{k,2}.sin(2\pi k f_{Mesh} t)\Big) + v(t)} \quad (5)$$

with

$$\tilde{a}_{k,l} = s_k\,cos(\xi_k + \psi_k(t)) \quad (6)$$

$$\tilde{b}_{k,l} = s_k\,sin(\xi_k + \psi_k(t)) \quad (7)$$

$$\tilde{c}_{k,l} = a_{k,l}.s_k.\cos(\theta_{k,l}).cos(\xi_k + \psi_k(t)) \quad (8)$$

$$\tilde{d}_{k,l} = a_{k,l}.s_k.\cos(\theta_{k,l}).sin(\xi_k + \psi_k(t)) \quad (9)$$

$$\tilde{e}_{k,l} = a_{k,l}.s_k.sin(\theta_{k,l}).cos(\xi_k + \psi_k(t)) \quad (10)$$

$$\tilde{f}_{k,l} = a_{k,l}.s_k.sin(\theta_{k,l}).sin(\xi_k + \psi_k(t)) \quad (11)$$

Which comes to:

$$s_f(t) = \sum_{k=1}^{K} \sum_{l=1}^{L} \tilde{R}_{k,l}.\tilde{U}_{k,l} + \tilde{T}_k.\tilde{w}_k + v(t) \quad (12)$$

with

$$\tilde{T}_k = [cos(2\pi k f_{Mesh} t) \quad -sin(2\pi k f_{Mesh} t)] \quad (13)$$

$$\tilde{U}_{k,l} = [\tilde{a}_{k,l} \quad \tilde{b}_{k,l} \quad \tilde{c}_{k,l} \quad \tilde{d}_{k,l} \quad \tilde{e}_{k,l} \quad \tilde{f}_{k,l}]' \quad (14)$$

$$\tilde{w}_k = [\tilde{w}_{k,1} \quad \tilde{w}_{k,2}]' \quad (15)$$

$$\tilde{R}_{k,l} = [\tilde{R}_{k,l}(1) \quad \tilde{R}_{k,l}(2) \quad \tilde{R}_{k,l}(3)] \quad (16)$$

$$\tilde{R}_{k,l}(1) = [cos(2\pi k f_{Mesh} t) \quad -sin(2\pi k f_{Mesh} t)] \quad (17)$$

$$\tilde{\boldsymbol{R}}_{k,l}(2) = \left[\cos\left(2\pi l f_p t\right).cos(2\pi k f_{Mesh} t) \quad -\cos\left(2\pi l f_p t\right).sin(2\pi k f_{Mesh} t)\right] \tag{18}$$

$$\tilde{\boldsymbol{R}}_{k,l}(3) = \left[sin\left(2\pi l f_p t\right).cos(2\pi k f_{Mesh} t) \quad sin\left(2\pi l f_p t\right).sin(2\pi k f_{Mesh} t)\right] \tag{19}$$

In terms of a matrix formulation, the above Eq.12 after discretizing with *t=nTs.*, where *Ts* is the sampling period the vibration signal, can be expressed as:

$$\underline{\boldsymbol{s_f}} = \boldsymbol{R_D}.\underline{\boldsymbol{U}} + \boldsymbol{R_A}.\underline{\boldsymbol{w}} + \underline{\boldsymbol{v}} \tag{20}$$

This model is linear in its coefficients.

The gear vibration signal is stored in vector $\underline{\boldsymbol{s_f}}$ such as $\underline{\boldsymbol{s_f}} = \left[\boldsymbol{s_f}(\boldsymbol{0}) \quad \boldsymbol{s_f}(\boldsymbol{1}) \dots\dots..\boldsymbol{s_f}(\boldsymbol{N})\ \right]$'

R_A is a *[N* x *2K]* matrix.

R_D is a *[N* x *2K.(2L+1)]* matrix.

$\underline{\boldsymbol{U}}$ *is* a [*2K.(2L+1)* x *1]* vector. It is considered as the signal of gear condition.

$\underline{\boldsymbol{w}}$ is a *[2K* x *1]* vector supposed to be an white Gaussian noise .

$\underline{\boldsymbol{v}}$ is a *[N x 1]* vector supposed to be an white Gaussian noise . $\underline{\boldsymbol{v}} = [\boldsymbol{v}(\boldsymbol{0}) \quad \boldsymbol{v}(\boldsymbol{1}) \dots\dots.\boldsymbol{v}(\boldsymbol{N})]$'

Let $\boldsymbol{s_{f,D}}$ is the modelled harmonic component of the vibration signal. This component carries the features induced by the gear tooth crack, while $\boldsymbol{s_{f,A}}$ is the unmodelled harmonic component of the vibration signal .

Eq.20 is re-written as follows:

$$\begin{cases} \underline{\boldsymbol{s_{f,D}}} = \boldsymbol{R_D}.\underline{\boldsymbol{U}} \\ \underline{\boldsymbol{s_{f,A}}} = \boldsymbol{R_A}.\underline{\boldsymbol{w}} \\ \underline{\boldsymbol{s_f}} = \underline{\boldsymbol{s_{f,D}}} + \underline{\boldsymbol{s_{f,A}}} + \underline{\boldsymbol{v}} \end{cases} \tag{21}$$

2.2 The signal of gear condition optimal estimation

An optimal linear filter is designed for reducing the noise level of the gear vibration signal. It is to be synthetized by minimizing the mean square error of the estimator $\underline{\hat{s}_{f,D}}$ of the component $\underline{s_{f,D}}$ such as.

$$\underline{\hat{s}_{f,D}} = F_1.\underline{s_f} + F_2.\underline{U} \tag{22}$$

where F_1 , F_2 two *[N* x *N]* matrices and $\underline{U}$ are to be determined from the following criteria:

$$\begin{cases} J_1 = E\left\{\underline{s_{f,D}} - \underline{\hat{s}_{f,D}}\right\} = 0 \\ \min_{F_1} J_2 = E\left\{\left\|\underline{s_{f,D}} - \underline{\hat{s}_{f,D}}\right\|^2\right\} \\ \min_{\underline{U}} J_3 = \left\|\underline{\hat{s}_{f,D}} - R_D.\underline{U}\right\|^2 + \alpha.\left\|\underline{U}\right\|^2 \end{cases} \tag{23}$$

The index J_1 means the estimator is unbiased.
The index J_3 can be solved by using least -squares method. The parameter α is well-known as the regularization parameter of an inverse problem.

By solving criteria of Eq.23, we obtain the following expressions for F_1 , F_2 and $\underline{U}$ **:**

$$\begin{cases} F_2 = (I - F_1).R_D \\ F_1 = R_A.R'_A.(R_A.R'_A + \beta.I)^{-1} \end{cases} \tag{24}$$

with $\beta = \frac{\sigma_v^2}{\sigma_w^2}$

and then

$$\begin{cases} ((\boldsymbol{F_1}.\boldsymbol{R_D})'(\boldsymbol{F_1}.\boldsymbol{R_D})' + \boldsymbol{\alpha}.\boldsymbol{I}).\underline{\widehat{\boldsymbol{U}}} =.(\boldsymbol{F_1}.\boldsymbol{R_D})'.\underline{\boldsymbol{s_f}} \\ \hat{\underline{\boldsymbol{s}}}_{f,\boldsymbol{D}} = \boldsymbol{F_1}.\underline{\boldsymbol{s_f}} + \boldsymbol{F_2}.\underline{\widehat{\boldsymbol{U}}} \end{cases} \tag{25}$$

3 Simulation-based validation of the proposed approach

The proposed method was tested on a synthetic dataset described below:

$$s_h(t) = \sum_{k=1}^{2}(\frac{2}{k}).cos\left(2\pi k.462t + \frac{\pi}{2k}\right) + v(t)$$

$$s_f(t) = \sum_{k=1}^{2}(\frac{2}{k}).\left(1 + \beta_k(t)\right).cos(2\pi k.462t + \frac{\pi}{2k} + \psi_k(t))$$

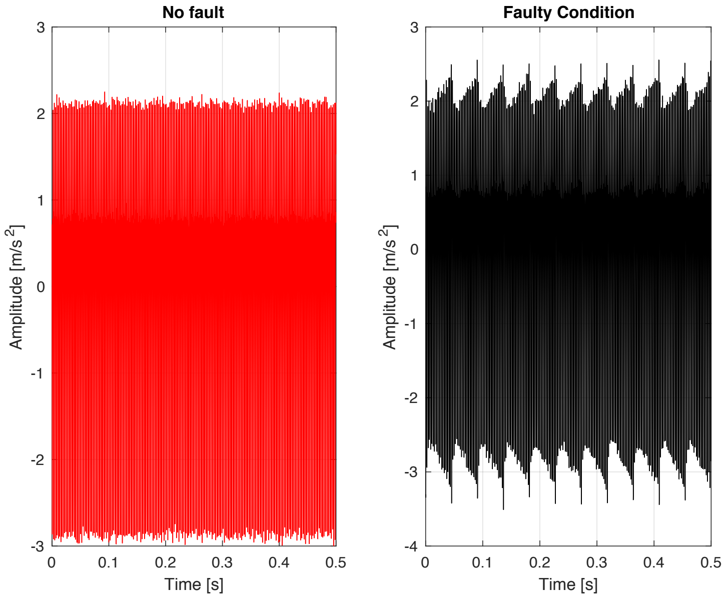
with
$\beta_k(t) = 1 + \sum_{l=1}^{10}\left(\frac{2}{l}\right).cos(2\pi l.22t + \frac{\pi}{3l}) + w_k(t)$ and $\psi_k(t) = \sum_{l=1}^{7}\left(\frac{2}{l}\right).sin(2\pi l.22t + \frac{\pi}{6l})$

For the generation of matrixes R_A and R_D, the different parameters were chosen as follows:
$K = 2,\ L = 5,\ T_s = 10^{-4},\ \beta = 10^{-10}$ and $\gamma = 10^{-7}$

Two levels of SNR have been used for testing the efficiency of the proposed approach to diagnose gear tooth crack. The SNR is defined as follows:

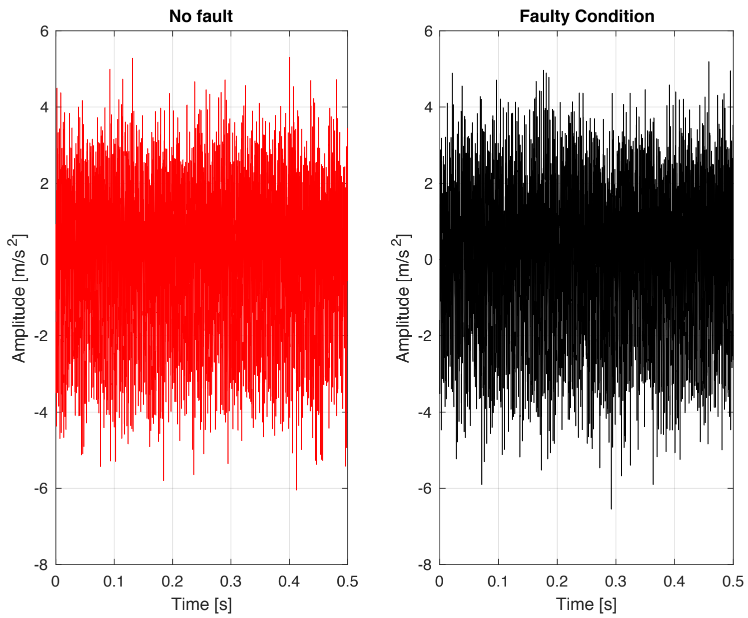
$$SNR = 10.log_{10}\frac{\sum_{n=1}^{N}(s_f(n))^2}{\sum_{n=1}^{N}(\mathrm{v}(n))^2}$$

The following figures described respectively the generated healthy and faulty signals simulated and the estimation of the modelled deterministic component for a SNR of 30 dB and 3 dB.



(a) (b)

Fig. 1 Gear vibration signal with a SNR of 30 dB a) non-faulty condition, b) faulty condition



(a) (b)

Fig. 2 Gear vibration signal with a SNR of 3 dB a) non-faulty condition, b) faulty condition

In Fig. 1b, with a SNR of 30 dB, the periodicity related to the localized fault is evident and the two health signals are different from each other. It's not the case for the lower SNR (3 dB in Fig. 2).

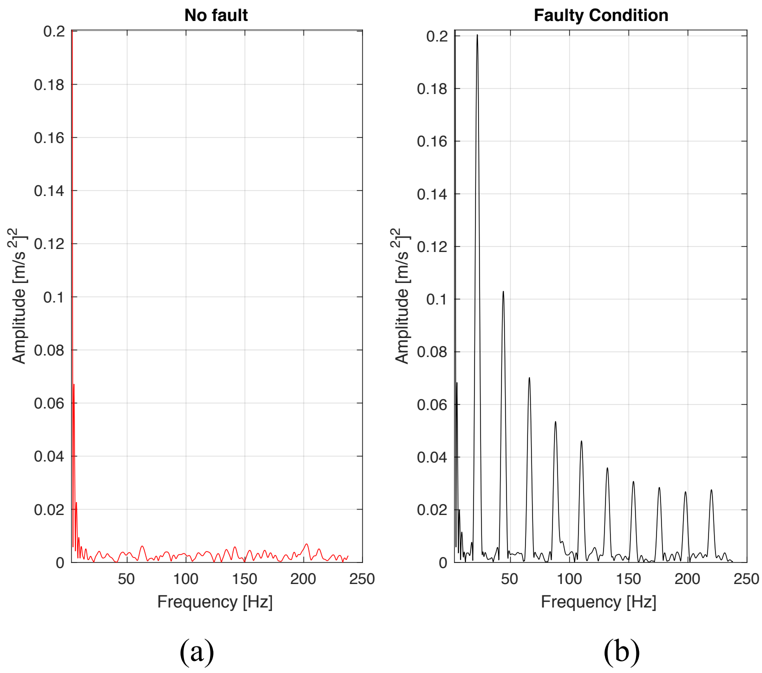


Fig. 3 Squared envelope of gear vibration signal with a SNR of 30 dB a) non-faulty condition, b) faulty condition

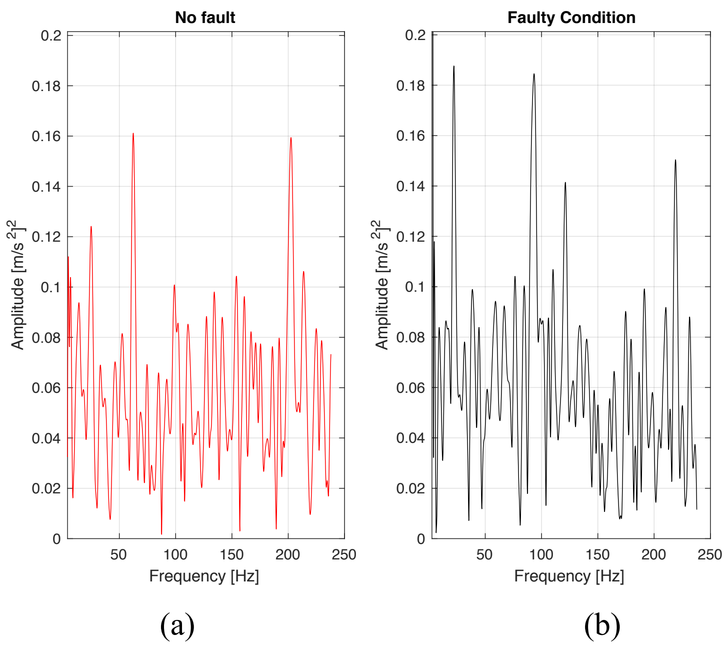
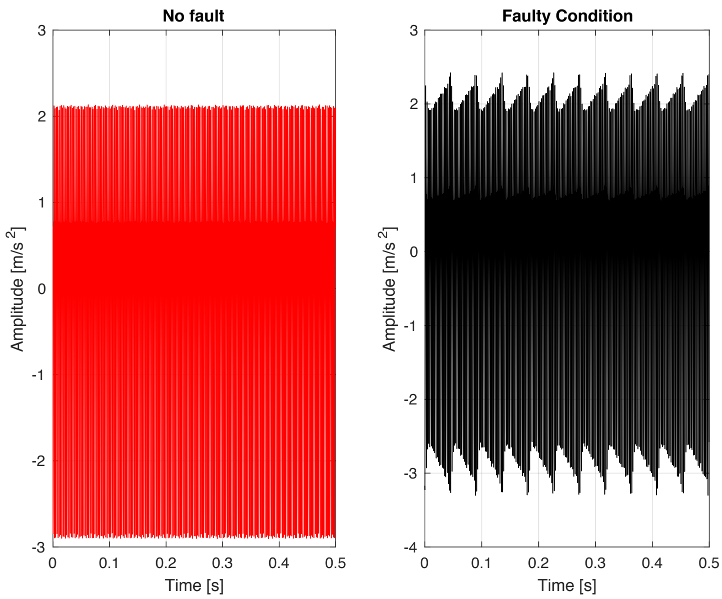


Fig. 4 Squared envelope of gear vibration signal with a SNR of 3 dB a) non-faulty condition, b) faulty condition

Figures 3 and 4 shows the squared envelope spectra for 30 dB and 3 dB SNR respectively. In Fig. 3 the fundamental fault frequency (22 Hz) and successive harmonics are quite evident. However, at low SNR the squared envelope spectra of the healthy and faulted condition are similar to each other and it is hard to discriminate between the two cases.

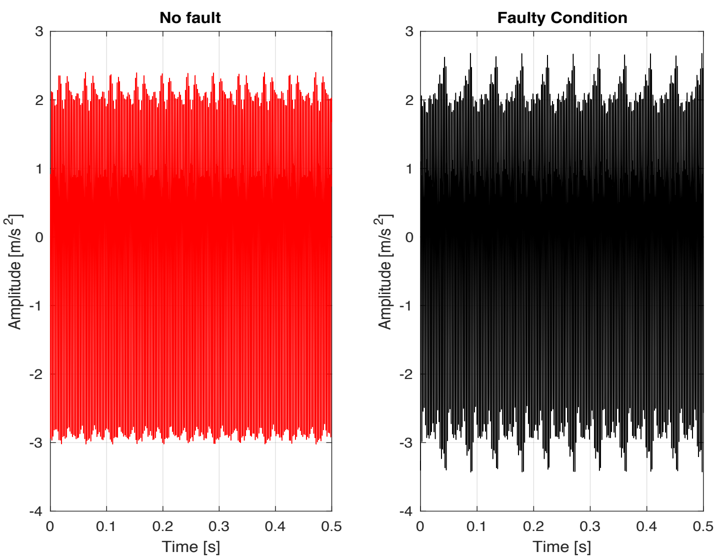
4 Results and discussion

In this section, the proposed diagnostic approach is validated through healthy and faulty vibration signal. In Fig. 5, the healthy and faulty estimated signal are similar to original ones, while in Fig. 6 a train of peaks is evident in both healthy and faulty conditions.



(a) (b)

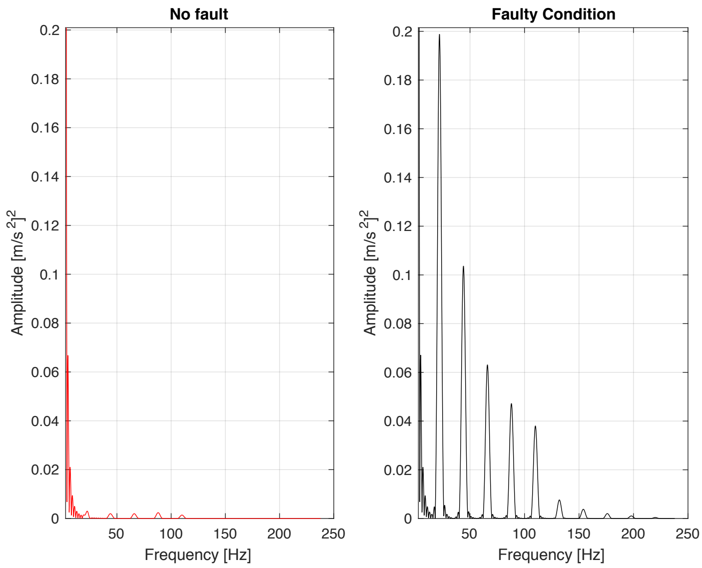
Fig. 5 Estimated of gear vibration signal with a SNR of 30 dB a) non-faulty condition, b) faulty condition



(a) (b)

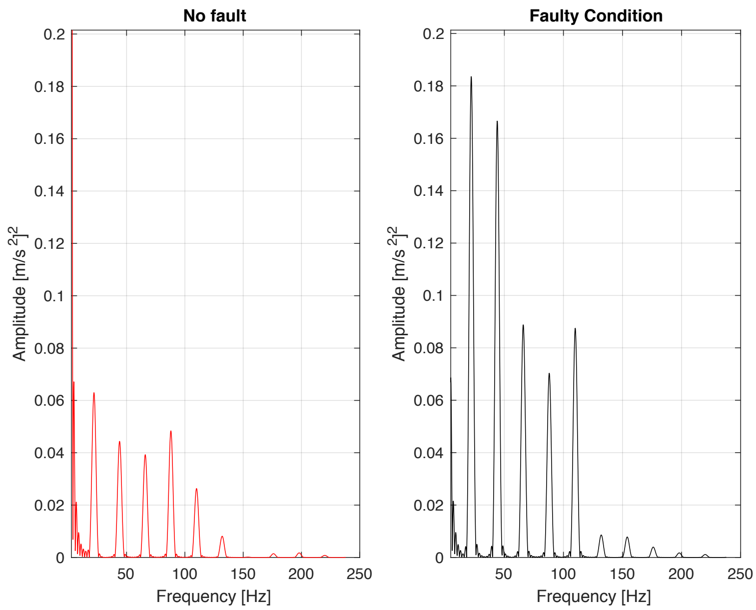
Fig. 5 Estimated of gear vibration signal with a SNR of 3 dB a) non-faulty condition, b) faulty condition

The squared envelope spectra of the estimated data are shown in Fig. 7 and 8. In Fig. 7b it is observed that there is a high-magnitude component at 22 Hz for the faulty case and then the magnitude of the other harmonics are decreasing like those of Fig. 3b. The healthy spectrum in Fig. 7a shows small peaks at the same frequency but their amplitude is negligible. The comparison between the two spectra at 30 dB SNR clearly highlight the presence of a fault in Fig. 7b.



(a) (b)

Fig. 6 Squared envelope of gear vibration signal with a SNR of 30 dB a) non-faulty condition, b) faulty condition



(a) (b)

Fig. 7 Squared envelope of gear vibration signal with a SNR of 3 dB a) non-faulty condition, b) faulty condition

In the case of 3 dB SNR (Fig. 8), the squared envelope of the healthy signal (Fig. 7a)it appears a carrier with its harmonics at the frequency of 22 Hz but with a lower amplitude. This is due to noise signal. Although the fault signal is very noisy, the same phenomena is observed in Fig.7b but amplitudes of carrier and its harmonics are considerably greater than those of healthy signal and so there is a clear evidence of the presence of the fault. In summary, the proposed approach achieves considerable noise reduction while preserving the failure-related signatures,

5 Conclusion

In this paper, a matrix formulation of the vibration signal was proposed for identifying the tooth crack in a gearbox system operating under constant speed conditions. Vibration signals acquired from gearboxes have noisy components that dominate, distorting the failure-related signal information, such as meshing frequency harmonics and distorting the failure-related signal information, To extract this information, a novel approach combining optimal filtering and deconvolution was proposed. This new approach significantly reduces the noise in the vibration signal while retaining failure-related information. Finally, the performance of the proposed methodology was evaluated using a real- simulated signal.

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