Time and angle analysis of Instantaneous Angular Speed signal: impact on average velocity and order spectrums

Extended Abstract

Arthur BUREL[1,2], Adeline BOURDON[1], Didier REMOND[1]
[1]Univ Lyon, INSA Lyon, CNRS, LaMCoS, UMR5259, 69621 Villeurbanne, France
[2]Safran Helicopter Engines, 64511 Bordes, France

1 Abstract

A one-degree-of-freedom rotating system is simulated under different periodic excitations, in order to extract its instantaneous angular speed (IAS) signal. A study of the corresponding spectrums in terms of time-domain and angle-domain frequencies shows a shift in peak locations from one domain to the other. This interrogates the constant speed assumption used for order spectrum analysis, and shows a distinction between angle-averaged and time-averaged velocity.

2 Introduction

2.1 Literature review

The measurement of signals on rotating machines in operation has been studied for over four decades for health monitoring applications, with a focus on the frequency content of accelerometer signals to detect periodic vibration phenomena, and separate normal excitation sources such as gear meshing from faults such as an unbalanced shaft or damaged gears or bearings. A common tool for time-based fault detection is the order spectrum, i.e. a frequency spectrum with its abscissa scale expressed in orders, multiples of the average rotation frequency of the shaft. Order tracking comes with the additional constraint of having a precise measurement for the rotating speed of the shaft, which can be difficult to achieve for variable speeds.
In such conditions, one may consider using signals sampled with a constant angle step, interesting for tracking faults with angular periodicity. The use of the IAS signal measured directly on the shaft, has the advantages of working in variable speed conditions like in wind turbines [1], and providing information directly coming from the rotor. IAS can be measured using the Elapse-Time method, counting the time between two angle steps on the wheel of an angular coder [2]. This angular sampling and the knowledge of the time-angle relationship allow the study of spectrums in angle-domain frequencies. Uncertainty on this signal due to sampling and quantization can be evaluated in similar ways to that of time-sampled signals [3].

2.2 Definitions

The present work, related to the analysis of signals collected on a rotating machine, uses some well-known notions in terms of digital signal processing (DSP) and spectral analysis. This specific contribution is made in the framework of angular approaches, where time sampling of the signals is partially replaced by angular sampling. Thus, some notions existing in the time domain, have their angle-domain counterparts. Angle-domain frequencies are defined as a number of events per angle unit (*rad^{-1}* in the I.S, or *rev^{-1}* in practice). They must not be mistaken with the I.S angular frequency of a wave defined in *$rad.s^{-1}$*, and not used here. Next, if a time-periodic (or harmonic) function has a constant time-domain frequency f in Hertz, an angle-periodic (or cyclic) function has a constant angle-domain frequency φ in events per revolution. Finally, a notion which will be discussed later, is that of angle-averaged velocity Ω_Θ, defined in its integral form by equation (1), as opposed to the time-averaged velocity Ω_T defined by equation (2). In the case of discrete signals, each integral can be replaced by a sum of ω values made at the *ad hoc* constant sampling interval.

$$\Omega_T = \frac{1}{t_B - t_A} \int_{t_A}^{t_B} \omega(\tau) d\tau \tag{1}$$

$$\Omega_\Theta = \frac{1}{\theta_B - \theta_A} \int_{\theta_A}^{\theta_B} \omega(\psi) d\psi \tag{2}$$

3 Methods

3.1 Dynamical system

The dynamical system illustrated in Figure 1 has been simulated, consisting of a disk driven to a certain average speed with a constant torque. The effect of a periodic excitation was studied, by applying a pure sine in torque with a constant time frequency f or a constant angular frequency φ. The dynamics of the system can be described by equation (3), derived from Newton's second law. I is the inertia of the disk, and α_θ is a damping coefficient.

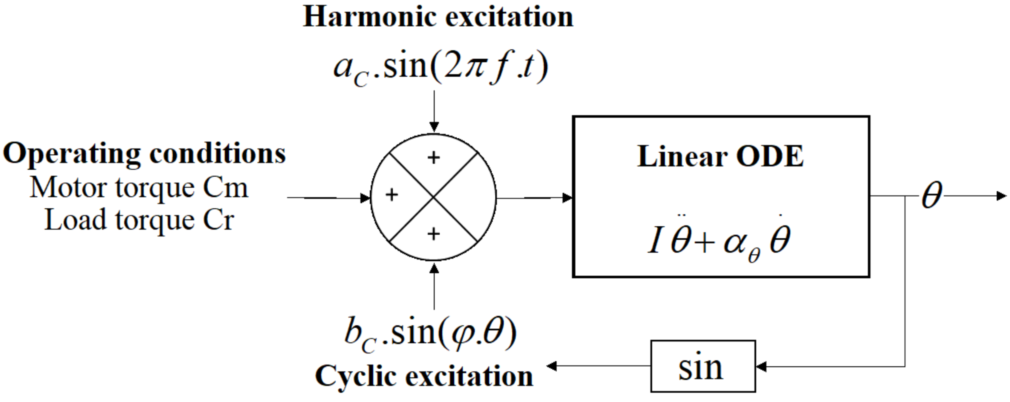

Figure 1: Dynamical block diagram of the studied system

In this paper, the choice is made to constrain the rotational degree of freedom by applying motor and resistant torques, C_m and C_r, which in the absence of excitation lead the first order velocity system to a stationary rotational speed ω_0 and the corresponding stationary rotational frequency f_0. In addition to these linear differential terms and constant torques C_m and C_r, the torque excitation term C_p is added, depending on both time and angle. The angle θ and its derivative ω are solved for in time steps using a fourth-order Runge-Kutta integration scheme, here with the ODE45 solver on Matlab. The ability to use θ as an input of the system along with time t comes with a weak coupling of the equations.

$$I.\ddot{\theta} = C_m - C_r - \alpha_\theta \dot{\theta} + C_p(t,\theta) \tag{3}$$

d (*mm*)	b (*mm*)	ρ ($kg.m^{-3}$)	I ($kg.m^2$)	α_θ ($N.m.s.rad^{-1}$)
84	20	7800	$7.62\ 10^{-4}$	0.0156

Table 1: Set of parameters defining the dynamics of the system

3.2 Digital signal processing

The instantaneous angular speed (IAS) ω, extracted from time integration, is resampled by interpolation, with either constant time or angle spacing, then its global average value is subtracted from it. Signals resampled with a uniform time scale lead to a spectrum in the domain of frequencies f, in Hertz. Alternatively, this frequency scale can be divided by the rotation frequency f_{ref} corresponding to a reference angular velocity Ω to get an order spectrum, where this reference frequency is the dimensionless unit. The third type of spectrum is obtained from signals sampled over a constant angle grid, where the abscissa is expressed in angular frequencies φ, with a number of events per revolution (rev^{-1}) as a unit.

4 Results and discussion

4.1 Order shift phenomenon under cyclic torque excitation

One attention point in frequency order analysis lies in the position of spectral peaks under cyclic velocity changes: It was previously shown that, for a cyclic excitation in torque, the peaks in the order spectrum based on the theoretical stationary rotation frequency f_0 were slightly different from the ones expected and obtained in the angular frequency spectrum [4]. In Figure 2, two definitions of the order spectrum are shown, differing by the reference velocity chosen. In the case of measurements made with an angular coder, the average speed used for the order scale would be calculated from samples made with constant angle step. Its value in the simulations is 490 revolutions per minute (51.31 radians per second), which is identical to the value ω_0. However, if we compute the average from the data resampled with constant time step instead, we obtain a value of only 461 revolutions per minute (48.24 radians per second). For four different excitation torques, it is shown that the ratio between the angle-averaged and time-averaged velocities is equal to the ratio between peak locations from one figure to the other: in fact, the excitation frequency corresponds to order 6.12 of the time-averaged, actual velocity, which is lower than the angle-averaged, theoretical velocity.

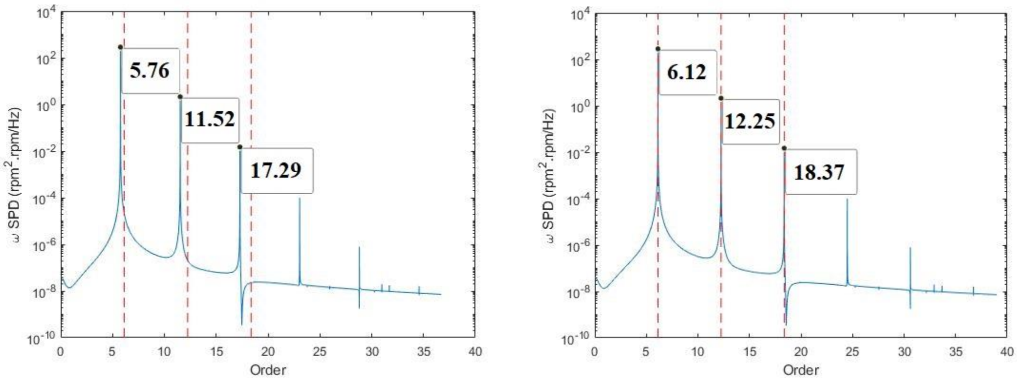

Figure 2: order spectrum for a cyclic excitation of 10% Cm at 6.126 events per revolution. Orders are based on angle-averaged velocity (left) and time-averaged velocity (right).

4.2 Explanation of the time and angle averaging bias

The difference between time-averaged and angle-averaged velocity finds its explanation in the difference between angle and time sampling. In the case of a pure sine excitation with a cyclic frequency of φ events per revolution, the velocity changes of the system have a null average over an angular period Θ of 2π/φ radians, but not over the equivalent time period T. A closer look at a cyclically excitated IAS signal is given in Figure 3, where the shape of the curves show an unbalance between high and low speed half-periods: for an average center value between peaks of 476 rpm (49.85 radians per second), the signal has larger time periods under than over this value, but larger angle periods over than under it, making the angle average equal to the stationnary velocity while the time average is lower.

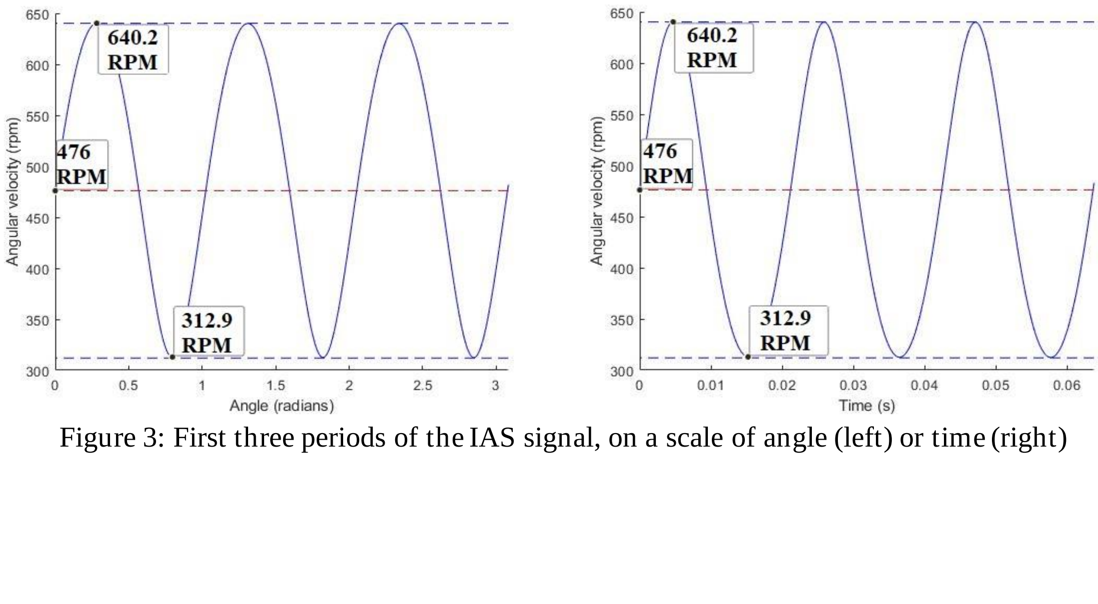

Figure 3: First three periods of the IAS signal, on a scale of angle (left) or time (right)

5 Conclusion

The present results show an interrogation point regarding the notion of average rotation speed. When exposed to a cyclic excitation of null average, the IAS of a rotating system keeps its stationnary average in the angle domain, but decreases in the time domain, due to the fact that slow angle intervals occupy more time than fast ones. The same phenomenon with a harmonic excitation, but instead the average rotation rate is unchanged in the time domain and increased in the angle domain. Although the simulated excitations are exagerated compared to real ones, the order shift would still be superior to available spectral resolutions. Finally, as IAS is directly linked to the time-angle relationship, it can be shown that these frequency shifts not only impact the IAS signal, but also possibly accelerometric signals at the base of the rotor.

6 References

[1] H. André, D. Rémond, et A. Bourdon, « On the Use of the Instantaneous Angular Speed Measurement in Non Stationary Mechanism Monitoring », ASME digital collection, juin 2012, p. 15-24. doi: 10.1115/DETC2011-47470.

[2] Y. Li, F. Gu, G. Harris, A. Ball, N. Bennett, et K. Travis, « The measurement of instantaneous angular speed », *Mechanical Systems and Signal Processing*, vol. 19, nº 4, p. 786-805, juill. 2005, doi: 10.1016/j.ymssp.2004.04.003.

[3] H. André, F. Girardin, A. Bourdon, J. Antoni, et D. Rémond, « Precision of the IAS monitoring system based on the elapsed time method in the spectral domain », *Mechanical Systems and Signal Processing*, vol. 44, nº 1, p. 14-30, févr. 2014, doi: 10.1016/j.ymssp.2013.06.020.

[4] A. Bourdon, D. Rémond, « Impacts de l'hypothèse de vitesse de rotation constante dans l'analyse et la modélisation du comportement de machines tournantes », *Congrès Français de Mécanique*, 2022.