Exploring the potential of sparse spectral estimation for vibration analysis

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Abstract

Recently, sparse signal processing algorithms have attracted much interest in the field of vibration-based machine monitoring as they leverage the assumption that most vibrations are sparse in at least one domain. While a large body of research focuses on the application of sparsity for gear and bearing fault detection in the time and envelope domain, this paper investigates the performance of a sparse data-adaptive spectral estimator as a precursor tool for other vibration signal pre-processing techniques. The Sparse Iterative Covariance-based Estimator or SPICE is a hyperparameter-free sparse spectral estimation method that has some promising statistical properties when compared to the standard Fourier transform. These characteristics make SPICE an interesting candidate for the analysis of complex vibration data coming from industrial machinery. This paper examines the efficacy of SPICE for vibration analysis by employing it for instantaneous angular speed estimation through integration with the multi-order probabilistic approach. Its advantages and shortcomings are explored and compared to the standard short-time Fourier transform as well as the short-time iterative adaptive approach. This new SPICE-based speed estimation approach is validated on realistic simulated data. The performance of the proposed methodology is then showcased on two complex wind turbine gearbox vibration data sets.

1 Introduction

Spectral analysis is a crucial component of many signal processing techniques, particularly in the field of rotating machine monitoring. This is due to the fact that the data sources commonly used in condition monitoring, such as vibrations, acoustics, speeds, and currents, capture the periodic nature of rotating machinery when sampled at sufficiently high frequencies. Spectral analysis enables these periodicities to be easily uncovered and interpreted. The Discrete Fourier Transform (DFT) is the most commonly used method for performing spectral analysis due to its ease of interpretation, lack of spectral line-splitting or frequency bias, non-parametric nature, and efficient computation via the Fast Fourier Transform (FFT). As a result, the DFT is a versatile tool for both experts and non-experts in industrial settings where critical decisions are often made with reliability in mind.

Although the DFT has these useful properties [1], particularly in regards to its ability to operate without a priori knowledge of signal models, there are some drawbacks associated with using it for complex rotating machinery. Specifically, the poor spectral resolution of the DFT limits its ability to resolve closely spaced spectral peaks, which can be problematic in certain applications. Additionally, the DFT tends to have high sidelobe levels and high variance, particularly when used on short measurements or in situations where the machine being studied produces low characteristic frequencies. To address these limitations, data windows can be employed to reduce sidelobe levels and variance in the DFT spectral estimate. However, selecting the most suitable window can be difficult, and each window has its own set of trade-offs [1]. In contrast to the DFT, parametric spectral estimation methods are available that can resolve close spectral peaks more effectively. However, these methods rely heavily on the assumption that the data contains a known number of sinusoids, which is rarely true when dealing with complex machinery [2].

Recently, there has been a growing interest in semi-parametric or sparse spectral estimation, which offers a promising alternative to the conventional Discrete Fourier Transform (DFT). These estimators are capable of producing higher resolution estimates than non-parametric methods, by making only weak assumptions such as sparsity [3]–[5]. In this paper, the potential of a sparse data-dependent approach called the sliding-window or short-time Sparse Iterative Covariance-based Estimator (ST-SPICE) is investigated [5], [6]. SPICE is a high-resolution spectral estimator that can reduce leakage effects with a trade-off of higher computational complexity. The study compares the performance of ST-SPICE with that of the short-time Iterative Adaptive Approach (ST-IAA) [7] and the short-time Fourier Transform (STFT) in the analysis of complex vibration signals. The main focus is on improving the estimation of the Instantaneous Angular Speed (IAS) while enforcing sparsity. Knowledge of the IAS is an often required precursor to further post-processing of vibration signals [8], [9].

This study discusses the use of the multi-order probabilistic approach (MOPA) [10] as a tool to compare spectral estimators and evaluate their potential for IAS estimation. While other IAS estimation methods exist, such as phase demodulation [11], MOPA employs time-frequency representations of the signal. Currently, MOPA uses the STFT to construct a probability-based 2D map of the IAS, which has been extensively tested and proven reliable. However, this study aims to investigate if replacing the STFT with a sparse spectral estimator could offer improved results while maintaining reliability and ease of use. Therefore, this work expands the so far limited application scope of these spectral estimators by investigating a more challenging experimental data source, i.e., wind turbine gearboxes.

2 Methodology

The short-time Fourier transform is the current staple analysis tool for non-stationary vibration signal processing thanks to it being a robust, non-parametric, and computationally efficient technique to analyze non-stationary signals through a time-frequency representation. However, despite the beneficial properties, the short-time Fourier transform also suffers from high variance, high sidelobes, and a low resolution. In previous work, the authors already investigated an alternative data-dependent spectral estimator, namely the iterative adaptive approach (IAA) [12]–[14]. In these studies, it was shown that significant accuracy improvements can be achieved by employing IAA whilst remaining hyperparameter-free. This study looks at a promising alternative to IAA that enforces sparsity of the spectral estimate to a greater extent.

2.1 Sparse Iterative Covariance-based Estimator

While IAA still returns a 'dense' spectrum, the Sparse Iterative Covariance-based estimator or SPICE typically produces a truly sparse spectrum with only a few Fourier coefficients being non-zero. SPICE itself is a spectral estimation technique that gained interest in the previous decade for the purpose of source localization, radar imaging, and missing data reconstruction [15]. So far however, SPICE has not been investigated on complex vibration data coming from a rotating machine. As such, it is unclear how it deals with the presence of a multitude of non-stationary harmonics in varying signal-to-noise ratios. While the original SPICE [5] is completely hyperparameter-free, this paper employs the generalized version, q-SPICE [15], as it allows for easy tuning of the sparsity and generally offers preferable estimates.

SPICE assumes that the vibration data adheres to the following linear model:

$$\boldsymbol{y}_N = \boldsymbol{B}_{N,K}\boldsymbol{\alpha}_K + \boldsymbol{e}_N \tag{1}$$

with $\boldsymbol{y}_N \in \mathrm{IR}^{\mathrm{N}}$ being the vibration signal of length N, $\boldsymbol{B}_{N,K} \triangleq [\boldsymbol{b}_N(\omega_0), \boldsymbol{b}_N(\omega_1), ..., \boldsymbol{b}_N(\omega_{K-1})]$ a dictionary matrix of regressors of size $(N\text{x}K)$, $\boldsymbol{\alpha}_K \triangleq [\alpha(\omega_0), \alpha(\omega_1), ..., \alpha(\omega_{K-1})]^T$ the complex-valued spectral amplitudes at the frequencies ω_k, and $\boldsymbol{e}_N$ an additive noise assumed to have zero mean and covariance matrix $\boldsymbol{\Sigma}$. If the signal's spectrum is sparse, this sparsity can be enforced on the vector $\boldsymbol{\alpha}_K$ as only a few of the amplitudes are assumed present in the signal. A common way to achieve this is through formulating the optimization problem as follows [16]:

$$\underset{\boldsymbol{\alpha_K}}{\text{minimize}} \;\; \frac{1}{2}||\boldsymbol{y}_N - \boldsymbol{B}_{N,K}\boldsymbol{\alpha_K}||_2^2 + \mu||\boldsymbol{\alpha_K}||_1 \tag{2}$$

where the first term penalizes the distance between the model and the signal, and the second term enforces sparsity upon $\boldsymbol{\alpha_K}$, with μ being an input parameter influencing the trade-off between the two terms. In [5], the original hyperparameter-free SPICE technique based on a covariance fitting criteria was proposed. The proposed minimization criteria was there formed as:

$$\underset{\tilde{\boldsymbol{p}}\geq 0}{\text{minimize}} \; ||\boldsymbol{R}_{\tilde{\boldsymbol{p}}}^{1/2}(\boldsymbol{R}_{\tilde{\boldsymbol{p}}} - \boldsymbol{y}_N\boldsymbol{y}_N^H)||_F^2 \tag{3}$$

with $||.||_F$ the Frobenius norm and $(.)^H$ the Hermitian transpose, and where

$$\boldsymbol{R}_{\tilde{p}} = \boldsymbol{A}\boldsymbol{P}\boldsymbol{A}^{\boldsymbol{H}} \tag{4}$$
$$\boldsymbol{A} = [\boldsymbol{B}_{N,K}, \boldsymbol{I}_{N,N}] \tag{5}$$
$$\boldsymbol{P} = diag(\tilde{\boldsymbol{p}}) \tag{6}$$
$$\tilde{\boldsymbol{p}} = [\boldsymbol{p}^T, \boldsymbol{\sigma}^T]^T \tag{7}$$
$$\boldsymbol{p} = [p_1 \dots p_K]^T \tag{8}$$
$$\boldsymbol{\sigma} = [\sigma_1 \dots \sigma_N]^T \tag{9}$$
$$\tag{10}$$

with $(.)^T$ denoting the transpose, $\boldsymbol{I}_{N,N}$ the N x N identity matrix, σ_n the noise variance for sample n, and $diag(\boldsymbol{v})$ the diagonal matrix of a vector $\boldsymbol{v}$. While Eq. 2 minimizes the distance between model and data, Eq. 3 employs the distance through the inverse of the covariance matrix. While Eq. 3 defines the original SPICE formulation, this study makes use of the later developed generalized SPICE or q-SPICE definition [15]. This definition allows for choosing the norm used for the noise parameters which enables adjusting the sparsity level and which circumvents the issue of the joint penalization of the signal and noise components of Eq. 3. The generalized SPICE can be expressed as follows [15]:

$$\underset{\boldsymbol{p}\geq 0, \sigma \geq 0}{\text{minimize}}\ \boldsymbol{y}^H\boldsymbol{R}^{-1}\boldsymbol{y} + ||\boldsymbol{W}\boldsymbol{p}||_r + ||\boldsymbol{W}_\sigma\boldsymbol{\sigma}||_q \tag{11}$$

where

$$\boldsymbol{W} = diag([w_1 \dots w_K]) \tag{12}$$
$$\boldsymbol{W}_\sigma = diag([w_{K+1} \dots w_{K+N}]) \tag{13}$$
$$w_k = \frac{||\boldsymbol{a_k}||_2^2}{||\boldsymbol{y}_N||_2^2} \text{ for } k = 1, ..., K+N \tag{14}$$

with $\boldsymbol{a_k}$ denoting the kth column of $\boldsymbol{A}$. As can be seen from Eq. 11, setting $r = 1$ and $q = 1$ produces the original SPICE definition. This study only considers the case where $r = 1$ and uses q to adjust the sparsity. The derivation of the generalized SPICE algorithm itself can be found in [15]. However, this naive implementation has a high computational complexity which encumbers the analysis of vibrations sampled at high frequencies. Therefore, an efficient SPICE implementation is employed based on the use of Gohberg-Semencul factorization for the case of a uniform noise variance [17]. By exploiting the Toeplitz structure of the covariance matrix $\boldsymbol{R}$, a speed up of two to three orders of magnitude can be achieved whilst reducing considerably the memory requirements.

2.2 Multi-order probabilistic approach

To illustrate the potential benefit in using SPICE in contrast to the DFT, this paper investigates using SPICE in a short-time manner and assesses SPICE's impact on instantaneous speed estimation using a state-of-the-art TFR-based approach, i.e., the multi-order probabilistic approach (MOPA). A concise summary of MOPA is presented here but interested readers are referred to [10], [18], [19] for more in-depth information.

The core concept behind the multi-order probabilistic approach (MOPA) initially proposed by Leclere et al. [10] revolves around treating the instantaneous spectrum of a vibration signal as a probability density function (PDF) of the Instantaneous Angular Speed (IAS). By analyzing the amplitude of the spectrum at a specific frequency, one can infer the likelihood of the shaft frequency of interest being equal to that frequency divided by a harmonic order, denoted as f/H_i, with H_i representing the i^{th} excitation or harmonic order.

To enhance the estimation of the IAS and make use of the information contained within the spectrum more effectively, it is necessary to incorporate multiple PDFs that correspond to different gear ratios or meshing orders. These individual PDFs can then be combined into a single PDF by multiplication. However, since the PDFs are independently generated for each time step, they do not ensure continuity of the IAS, which is a characteristic of mechanical systems. In practice, sudden and substantial acceleration or deceleration jumps in the IAS are unlikely due to the inertia of the rotating shafts. Hence, an assumption of continuity is introduced for the IAS.

The proposed approach involves generating a PDF for each time step based on the PDFs from several preceding and subsequent time steps. The appropriate weighting of these PDFs is achieved by convolving them with a centered Gaussian function, while the time relationship is incorporated by varying the variance based on the time interval between the considered PDF and the central PDF. The parameters of the centered Gaussian can be determined based on the maximum expected acceleration of the IAS. The resulting two-dimensional PDF map can then be utilized to extract the estimated IAS, which is obtained by calculating the expected value over time within a predefined range of IAS values. MOPA has demonstrated its reliability and accuracy as an IAS estimation tool by successfully considering multiple harmonic orders simultaneously.

3 Simulation results

To investigate the performance of the short-time SPICE and compare it to the conventional STFT and the ST-IAA, a simple vibration signal is simulated consisting of 10 non-stationary sinusoids corrupted by an additive white Gaussian noise with a signal-to-noise ratio of 20 dB, with the signal of interest being the non-stationary sinusoids and the noise the white Gaussian noise. The signal itself is 40 secs long and sampled at 100 Hz. The IAS of interest here is that of the base harmonic or fundamental which increases slowly for the first 10 secs, leading to closely-spaced harmonics, and then quickly for the next 10 secs. The IAS reverses course for the last 20 seconds. Figure 1 displays the time-frequency representation (TFR) of the simulated signal after processing with the STFT, ST-IAA, and ST-SPICE. The same color scale and input parameters are used for all three TFRs, i.e., a rectangular window size of 0.9 secs, 95% overlap, and an FFT size of 14 secs. As can be observed, the ST-SPICE TFR is much sparser than that of the STFT and ST-IAA, which is to be expected due to SPICE's sparsity enforcement. In this case it also sharpens the visual assessment of the main signal content, i.e., the 10 harmonics, since there is essentially no noise visible in the background. Despite this visual improvement, the real question is whether it actually can improve the performance of a vibration analysis method, in this case IAS estimation through MOPA.

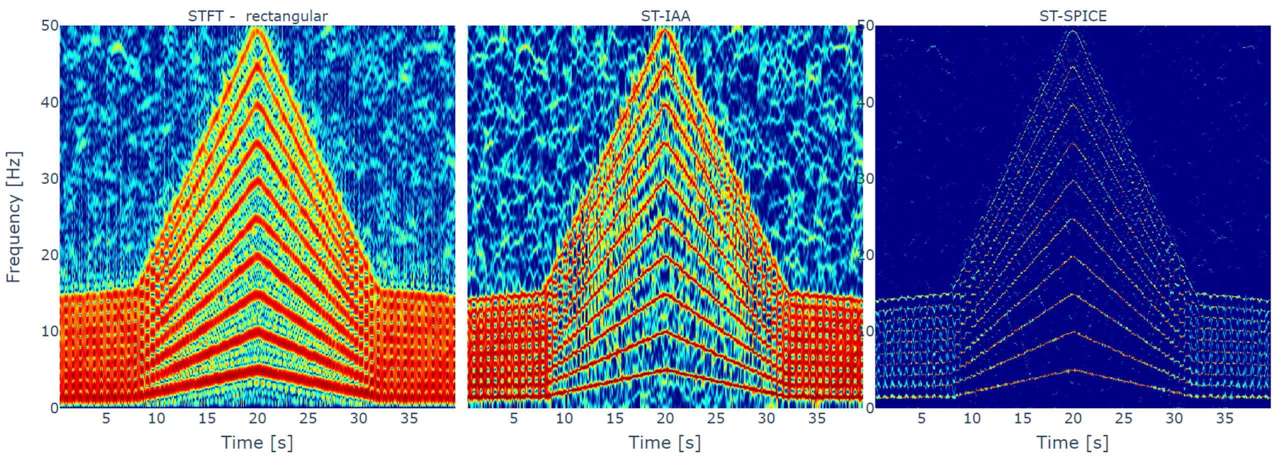

Figure 1: Time-frequency representations (TFRs) of the simulated signal using (left) STFT, (middle) ST-IAA, (right) ST-SPICE.

To assess this potential performance gain, the TFRs of Fig. 1 are fed to MOPA which results in the IAS estimates shown in Fig. 2. It should be noted that the same input parameters are employed by MOPA for all three TFRs, viz. the only differences between the IAS estimates originate from the employed TFR estimation technique. Based on Fig. 2, it can be seen that the STFT-based IAS estimate struggles with properly utilizing the low-frequency, closely-spaced harmonics, which is something ST-SPICE manages significantly better for this simulation case. While it is difficult to assess the overall estimation accuracy purely from the IAS profiles in Fig. 2, the mean and median absolute absolute errors are displayed in Fig. 3. As expected based on the estimated IAS profiles, ST-SPICE substantially outperforms the STFT with regard to accuracy of the IAS estimate and is also slightly more accurate than the ST-IAA. The next section investigates whether this same observation can be made on more complex, real-world use cases.

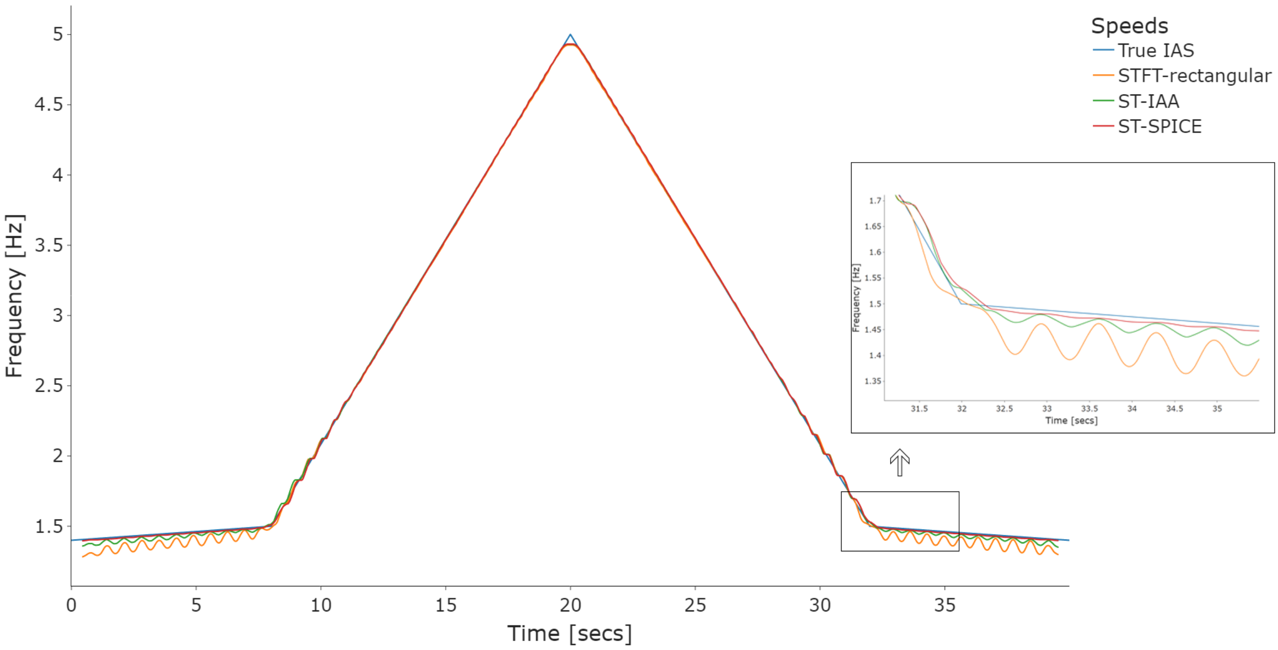

Figure 2: IAS profiles estimated by using MOPA with the TFRs of STFT, ST-IAA, and ST-SPICE on the simulated data. A zoom of the low-frequency region is also provided.

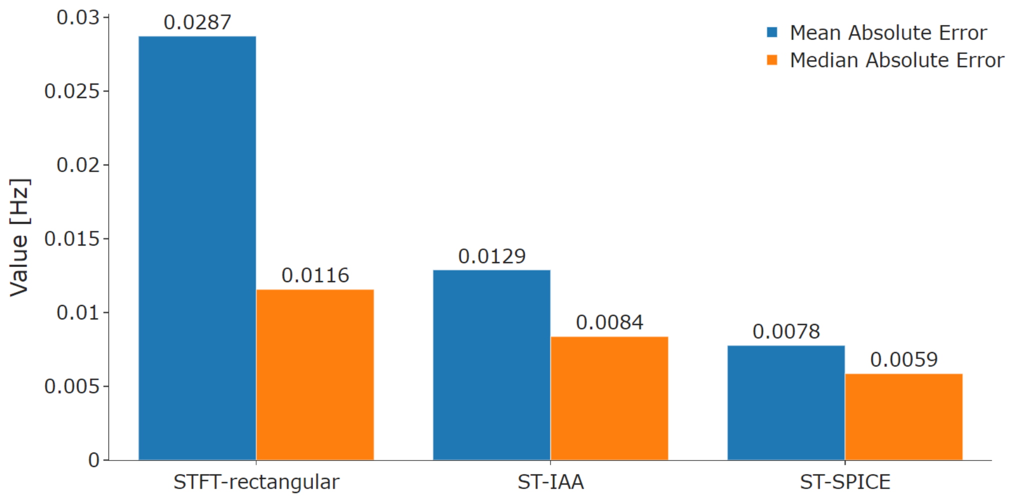

Figure 3: Mean and median absolute errors of the IAS estimation per TFR method.

4 Experimental applications

This section inspects the performance of ST-SPICE on experimental vibration data originating from measurements on in-the-field gearbox housings of large wind turbines. Due to confidentiality, the authors cannot disclose the exact details of these wind turbines however. The first case focuses on the performance of the three discussed TFR estimators for very low frequency signal content (<1 Hz). The second case on the other hand analyzes the accuracy when the sample rate is higher but there are closely-spaced harmonics.

4.1 Wind turbine gearbox case #1

This case investigates a vibration signal measured near the low-speed stage of a gearbox housing of an onshore multi-megawatt wind turbine. The sample rate was 2 Hz and the duration over 400 secs. One of the strongest vibration harmonics in most three-bladed wind turbines is the third harmonic of the rotor speed or 3P harmonic. This 3P frequency can be clearly seen in all three TFRs shown in Fig. 4. Given the low sample rate, there are not many clearly distinguishable harmonics present in the signal, in fact, in this case only the 3P harmonic has a decent SNR. Using MOPA on this data set thus boils down to smoothing in time of the expected frequency profile for the displayed TFRs in Fig. 4. The resulting IAS profiles are displayed in Fig. 5.

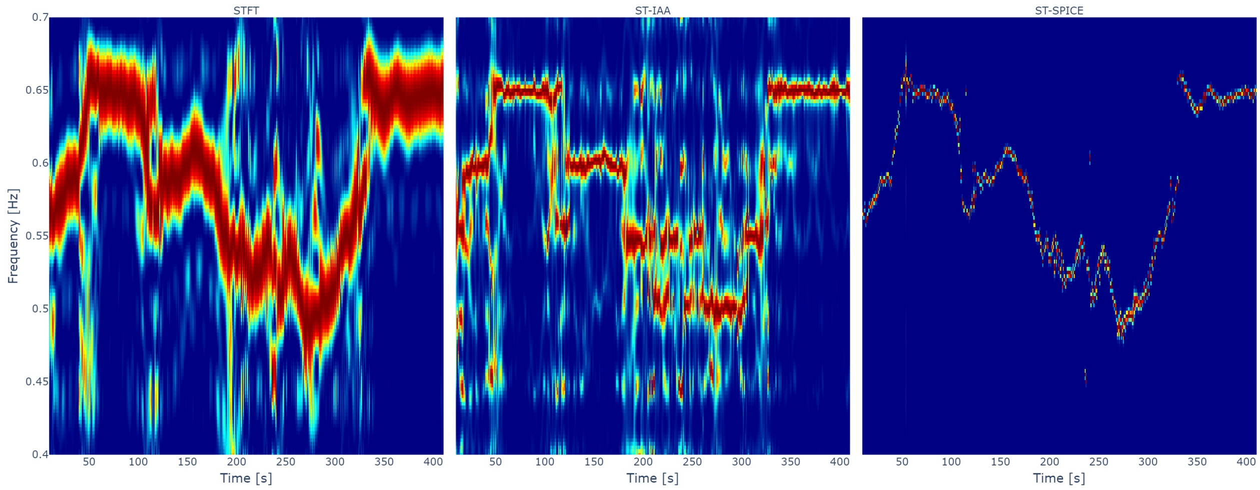

Figure 4: Time-frequency representations (TFRs) of the experimental vibration signal of wind turbine case #1 using (left) STFT, (middle) ST-IAA, (right) ST-SPICE.

Clearly, the IAS profiles in Fig. 5 showcase some strong deviations from the true speed, which is to be expected due to low frequency content and lack of higher harmonics. However, visually, the ST-SPICE IAS profile manages to approximate the true IAS better than the other two methods. This is confirmed by the mean and median absolute errors shown in Fig. 6. Again, the STFT is the least accurate with ST-SPICE being the most accurate. This case showcases the strength of SPICE as a sparse spectral estimator when the frequency resolution is insufficient to get a good peak estimation through the conventional DFT.

4.2 Wind turbine gearbox case #2

This second case analyses the performance of SPICE on complex multi-harmonic wind turbine data. The data comes from another large multi-megawatt wind turbine, but is this time sampled at 100 Hz and has a length of approximately 100 secs. This data was measured on the high-speed stage of the gearbox housing and exhibits the harmonics related to the high-speed shaft as well as gear meshing frequencies of the other stages. Additionally, the data contains several resonances and amplitude modulations corresponding to the different rotating components. Figure 8 features the TFRs for the three different methods, again with identical input parameters. Similar to the simulation and previous turbine case, ST-SPICE has a very fine energy concentration in just a few Fourier coefficients for the dominant harmonics.

Different from case #1, this measurement does allow for employing multiple harmonics due to its higher sample rate. Thus, MOPA uses 10 different harmonics of the fundamental harmonic of the high-speed shaft, based on the gearbox kinematics. Again the same MOPA input parameters are used by all three methods. The resulting TFR-based IAS estimates along with the encoder-based IAS estimate can be seen in Fig. 8 and the

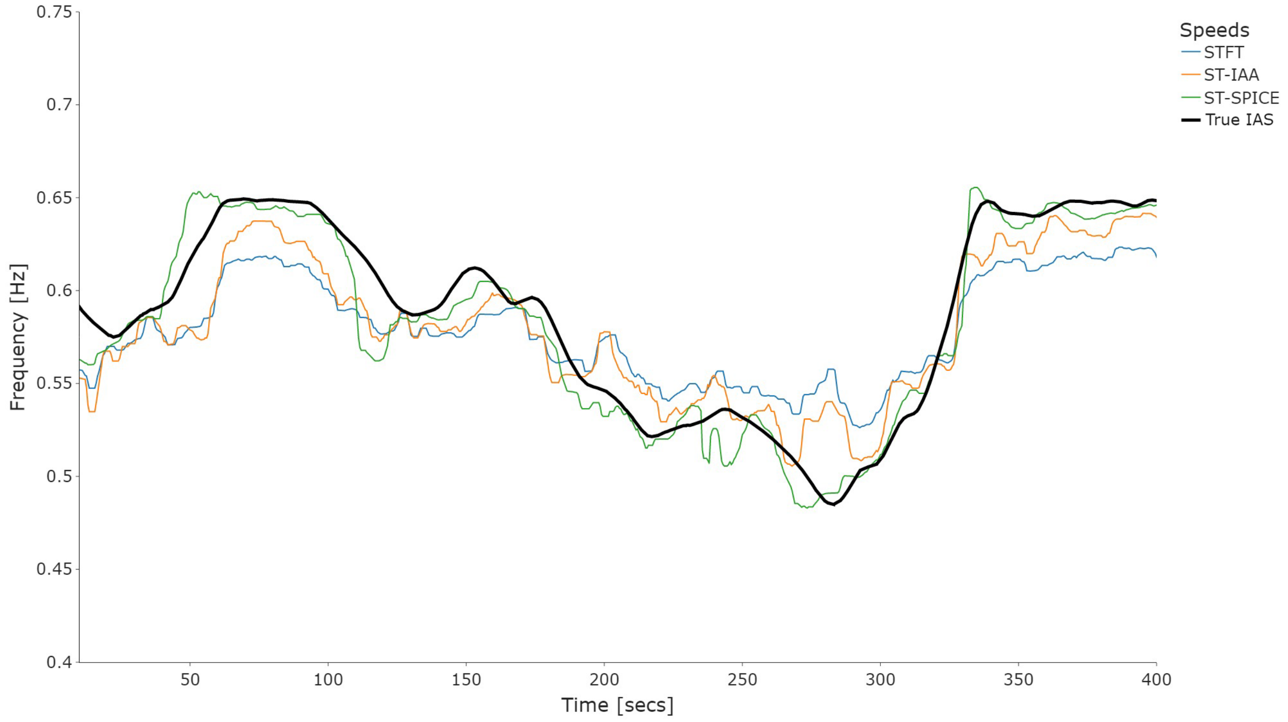

Figure 5: IAS profiles estimated by using MOPA with the TFRs of STFT, ST-IAA, and ST-SPICE on the experimental vibration signal of wind turbine case #1. The true IAS represents the estimate provided by a high-resolution angle encoder.

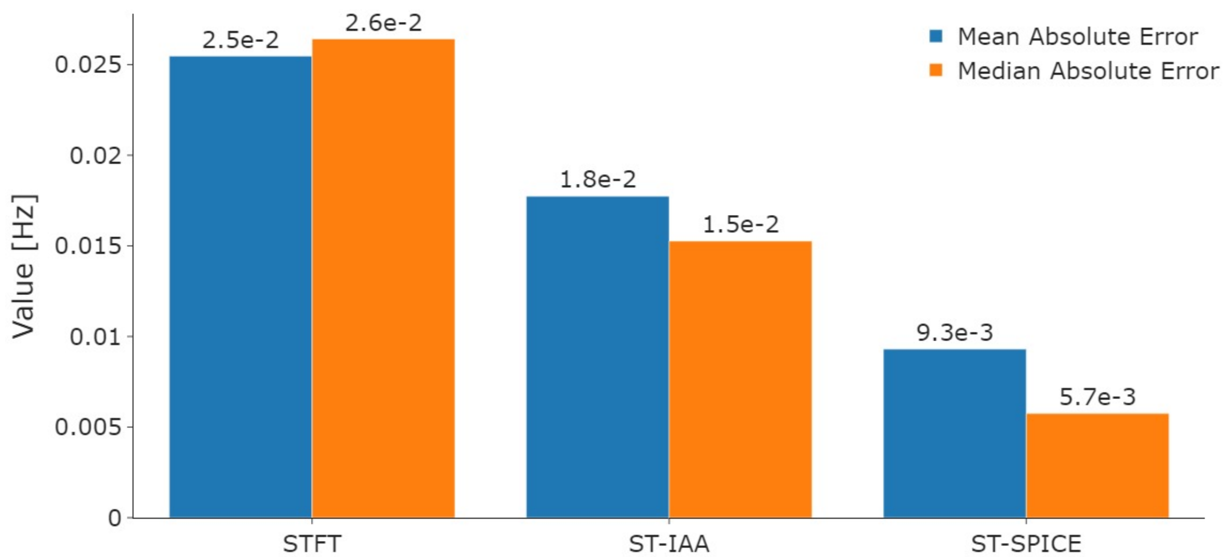

Figure 6: Mean and median absolute errors of the IAS estimation per TFR method for the experimental vibration signal of wind turbine case #1.

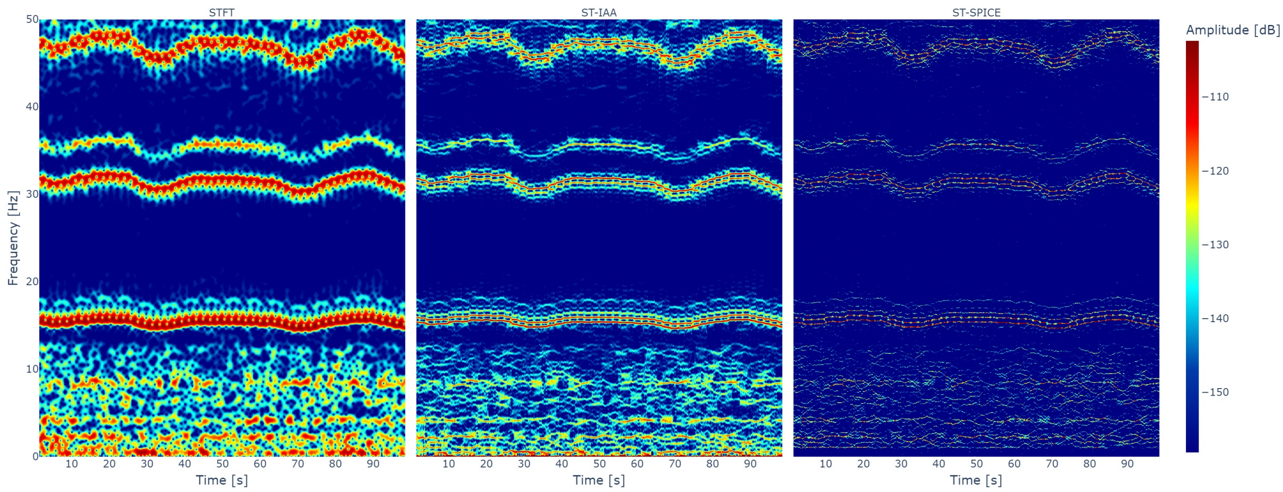

Figure 7: Time-frequency representations (TFRs) of the experimental vibration signal of wind turbine case #2 using (left) STFT, (middle) ST-IAA, (right) ST-SPICE.

corresponding mean and median absolute errors of these IAS estimates in Fig. 9. The findings of the previous analyses are confirmed by Fig. 9 for this experimental case as well, namely that the ST-SPICE can outperform the conventional STFT for real-world use cases. Additionally, the ST-IAA provides an adequate alternative having an accuracy that is nearly on par with that of ST-SPICE. The former can be especially useful in case there are only low SNR harmonics present in the signal, as the sparsity enforcement by SPICE can lead to suppression of very weak harmonics.

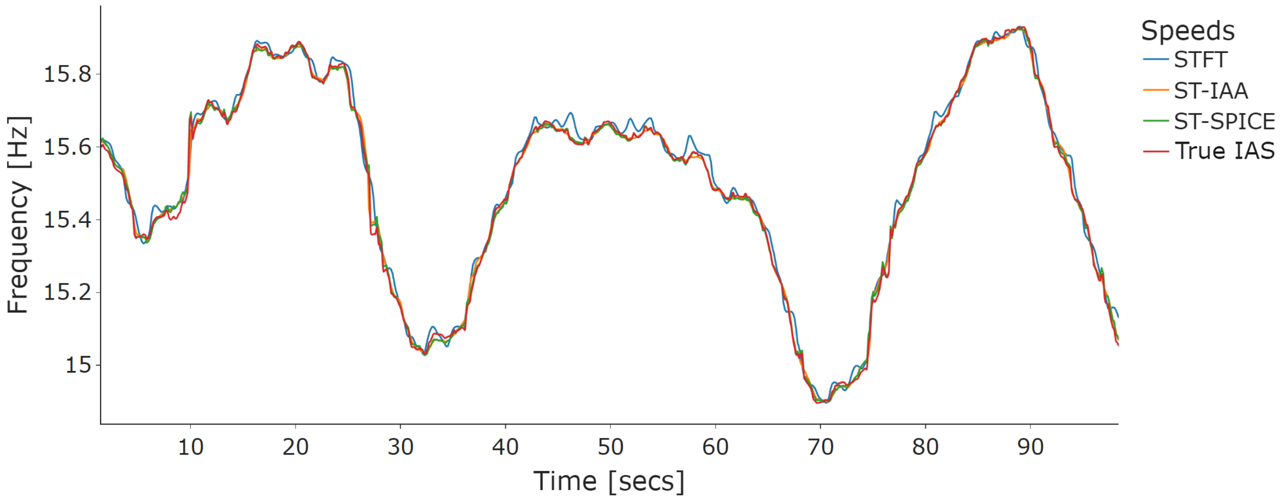

Figure 8: IAS profiles estimated by using MOPA with the TFRs of STFT, ST-IAA, and ST-SPICE on the experimental vibration signal of wind turbine case #2. The true IAS represents the estimate provided by a high-resolution angle encoder.

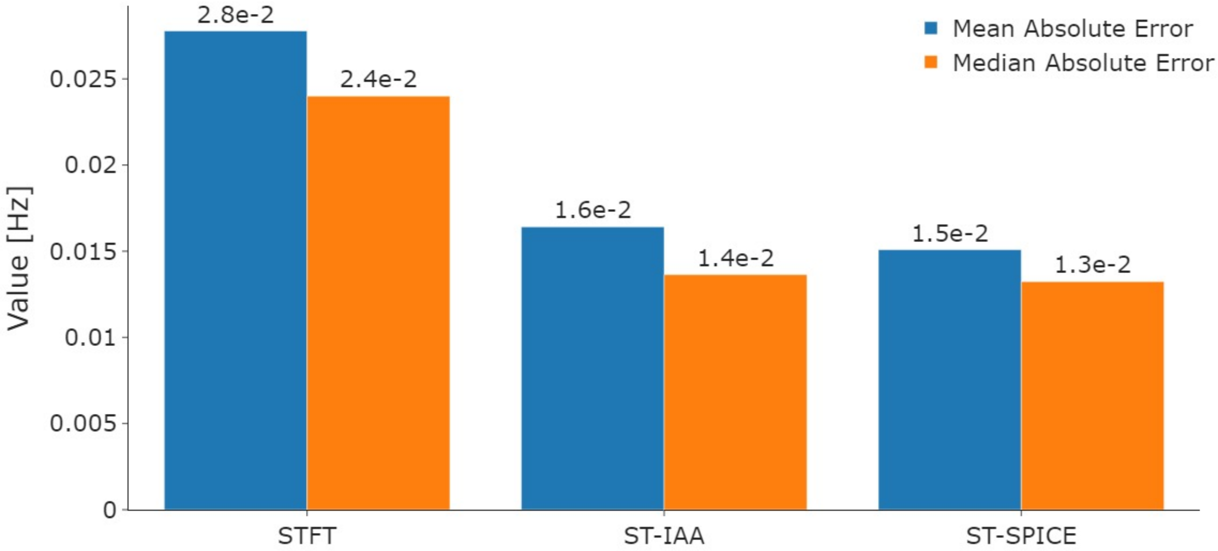

Figure 9: Mean and median absolute errors of the IAS estimation per TFR method for the experimental vibration signal of wind turbine case #2.

5 Conclusion

This paper investigates the potential benefit of using semi-parametric or sparse spectral estimators for complex vibration signal analysis. Specifically, it examines the hyperparameter-free SParse Iterative Covariance-based Estimator or SPICE and its capability for providing accurate estimates of harmonic frequencies in vibration signals. While SPICE incurs a much larger computational cost than the Fast Fourier Transform or FFT, it does however drastically reduce leakage effects without the need for subtle parameter tuning which is beneficial for industrial vibration analysis schemes. This study compares the performance of a short-time version of SPICE to that of the standard short-time Fourier transform (STFT) and a short-time Iterative Adaptive Approach (ST-IAA) for the purpose of instantaneous angular speed estimation. The latter method, IAA, is another semi-parametric spectral estimator which offers similar benefits to SPICE. A simulated case study and two experimental case studies confirm that SPICE can provide a real-world performance difference for vibration monitoring pipelines when accurate knowledge of harmonic frequencies is valuable. Especially for use cases where the harmonics of interest are located at very low frequencies or are closely spaced, SPICE offers a valuable advantage over the conventional discrete Fourier Transform.

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