Estimation of the piezoelectric factor in nonlinear transducers

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Abstract

This study focuses on the coupling characterisation of a structure displaying a geometric elastic nonlinearity and paired with piezoelectric elements. In the case of linear transducers, several equivalent definitions based on electrical and mechanical parameters are used to estimate the electromechanical coupling. The validity of these formulations are questioned in the non-linear case, which requires to reconsider the definition of the coupling factor.

The definition based on energy transfer inside the resonator is extended to monitor its evolution with the dynamic amplitude of the device. It is supposed that the coupling would not only be function of the equivalent parameters but also of the solicitation amplitude. The definition based on the natural frequencies is also extended to exploit the backbones curves of the transducer under different electrical configurations. An experimental study is performed an on a clamped-clamped beam covered with piezoelectric patches to compare the results obtained with these two methods.

1 Introduction : linear definition of the coupling factor

The elaboration of piezoelectric transducers is based on the maximization of their coupling factor to ensure optimal performances. In the case of linear piezoelectric transducers, several physical interpretations of the coupling factor coexist and numerous equivalent equations can be employed [1].

The coupling factor k can be interpreted as the capacity of the transducer to convert mechanical energy into electrical energy and vice-versa. It represents the portion of input energy converted and stored by the system and according to Manson [2] can be defined as :

$$k^2 = \frac{converted\ mechanical\ energy}{input\ electrical\ energy} = \frac{input\ electrical\ energy}{input\ mechanical\ energy} \quad (1)$$

This literary definition can be applied to a quasi-static effort cycling to be measured experimentally [3]. The transducer is first maintained in open-circuit (OC) while an increasing displacement is applied up to the chosen value d_{max}. Once reached, the electrodes are connected and the prescribed displacement is decreased while the transducer remains in short-circuit (SC) as presented on Figure 1.

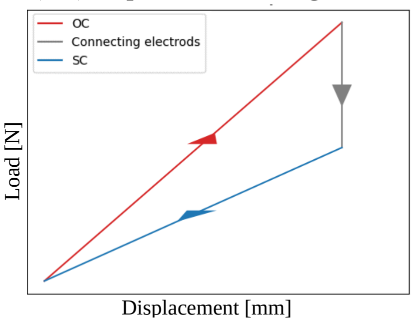

Figure 1:Theoretical CO-CC cycle

Considering the energies U relative to the two steps of this cycle, the coupling factor can be expressed as:

$$k^2 = \frac{U_{oc} - U_{sc}}{U_{oc}} \quad (2)$$

An other physical interpretation has been proposed by Hunt [4], where the coupling factor can also be linked to the change in mechanical impedance produced by the electromechanical coupling. The values of the stiffness K varies with the electrical boundary conditions, and the change between SC and OC can be exploited to derive the coupling factor:

$$k^2 = \frac{K_{oc} - K_{sc}}{K_{oc}} \quad (3)$$

In the linear case, equation (3) is directly equivalent to equation (2). Moreover, supposing that the modal shape is similar in OC and SC, the coupling factor can be derived from the natural frequency f relative to these electrical states as presented on Figure 2:

$$k^2 = \frac{f_{OC}^2 - f_{SC}^2}{f_{OC}^2} \quad (4)$$

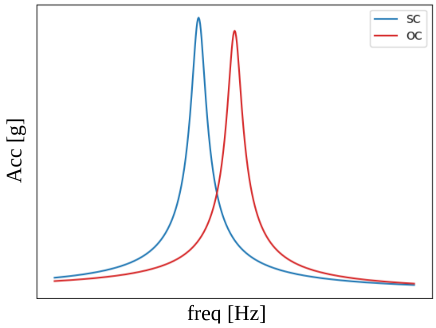

Figure 2: Frequential response of a linear piezoelectric transducer in Sc and OC

In the linear case, these different equations are equivalent and static or dynamic methods can be applied arbitrarily to deduce the value of the coupling factor.

2 Extension of the coupling definition for non-linear transducers

This paper focuses on the study of a non-linear piezoelectric transducers : piezoelectric elements are disposed on a mechanical structure displaying a geometric non-linearity. In the non-linear case, the validity of the equation based on stiffness variation is no longer ensured [1] and the equivalences between the different formulations do not apply. This arises the question of the formulation to employ for a piezoelectric transducer composed of a base structure displaying mechanical non-linearity.

Wu [5] proposed a numerical comparison of three formulations based on the electrical energy stored in the equivalent capacitance, the natural frequencies in OC and SC and the remarkable frequencies of the electrical admittance. Two piezoelectric cantilever beams were employed with the addition of two types of nonlinearity: a frictional contact at its extremity or a junction to a high-order nonlinear stiffness. From his validation protocol, only the energy approach appeared as applicable to develop a nonlinear modal electromechanical coupling factor.

In this study, it is proposed to extend the energy based definition of Manson via the application of a quasi-static cycling under different electrical configurations. For comparison, the approach based on the natural frequencies of the system in OC and SC is also considered. In that case, the formulation of the coupling defined by (4) is based on the evolution of the equivalent frequencies, i.e. the backbones relatives to the two electrical boundary conditions. An experimental set-up was implemented to compare the obtained results of the two approaches. A thin aluminium beam is clamped at its two extremities and two piezoelectric ceramics are glued on one surface.

As the effort-displacement relation of the two steps of the proposed cycling is not linear and the equivalent frequency is not constant, the coupling factor of a nonlinear transducer will vary with the amplitude of solicitation.

2.1 Energy approach: quasi-static cycling

A first methodology based on a mechanical quasi-static cycling in open and closed circuit has been developed to estimate the coupling factor for a given solicitation. A loading and unloading cycle was performed while switching the electrical condition between the two steps. This study was performed by prescribing the displacement at the centre of the beam. To compare the obtained coupling factor, the two cycles OC-SC and SC-OC were implemented. Cycles were performed for increasing solicitation levels to visualize their evolution.

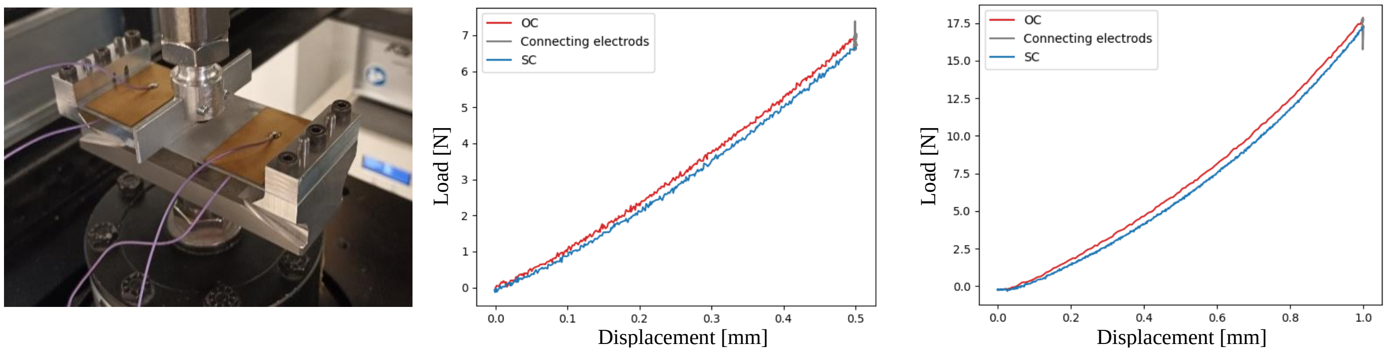

Figure 3: Quasi-static cycles OC-SC, (a) Experimental set-up, (b)Imposed displacement of 0.5mm, (c)Imposed displacement of 1mm

As can be seen on Figure 3, the displacement-effort relations in SC and OC are impacted by the value of the imposed maximal displacement. A numerical model has been developed to better apprehend the portion of mechanical and electrical energies to considerate, in order to understand the impact of the geometric non-linearity on the piezoelectric coupling.

2.2 Frequency approach : dynamic solicitation

A second dynamical experiment has been conducted to extend the equations based on the OC and SC frequencies of the system. The piezoelectric beam is mounted on a electrodynamic shaker table to assess the evolution of the response spectra and the equivalent frequency with the amplitude solicitation.

As can be seen on Figure 4, two distinct backbones corresponding to the OC and the SC cases can be depicted by this method. The system presents a softening and then hardening response and the nonlinearity results in a sudden jump of the frequency response for large amplitudes of solicitation. The evolution of the coupling factor derived from equation (4) can thus be computed after the extraction of the equivalent and jump frequencies along the backbones.

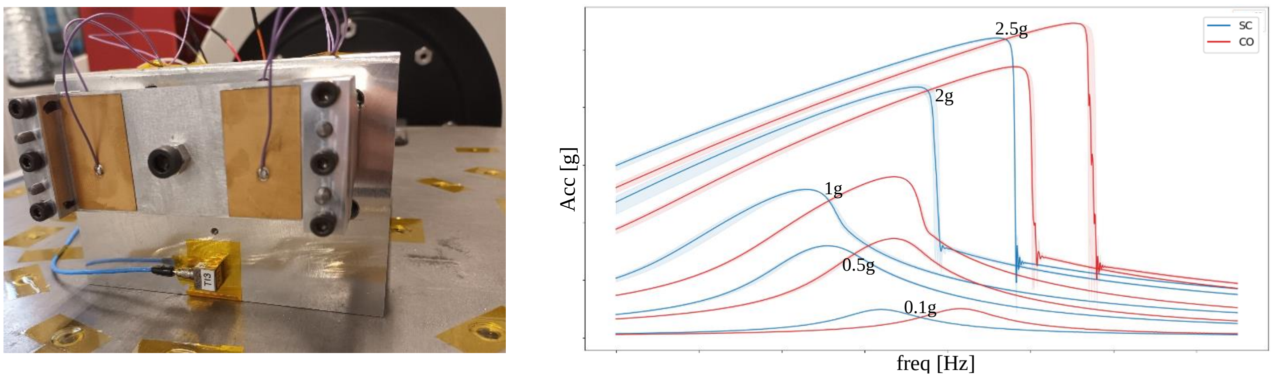

Figure 4: Evolution of the frequency spectra in SC and OC, (a) Experimental set-up, (b) Mean curves obtained for various amplitude of solicitation

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