Control based continuation of autonomous system — Stabilization mechanisms

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1   Introduction

Experimental analysis of nonlinear dynamical system is always a complicated task because these system can exhibit bifurcations, multi-stability or unstable behavior. When a mathematical system is available, numerical continuation algorithm constitute a powerful tool to investigate the so-called bifurcation diagram. Sieber and Krauskopf introduced the idea of control based continuation, obviating the need of a mathematical model so that it can be used to obtain the bifurcation diagram from a physical experiment [1]. Control based continuation methods rely on a feedback controller to stabilize the possibly unstable response of the system under test. This feedback controller must be non-invasive to ensure that the solution of the controlled system is also a solution of the uncontrolled system.

Since then, control based continuation received a growing interest and has been mostly used to track branches of periodic solutions of non-autonomous (i.e. forced) systems [4, 3, 2]. Although it has already been applied on various experiments, only few papers deal with the detailed analysis of the underlying stabilization mechanism [5]. Applying these methods to autonomous systems presents an additional difficulty since the frequency of the limit cycle is also an unknown. Numerical continuation algorithms solve this problem by appending the system with a phase condition.

The objectives of this study are twofold. (i) design a controller able to stabilize limit cycle oscillations, (ii) investigate the underlying stabilization mechanism of the designed controller.

2   Results and discussion

The proposed control strategy consists in coupling a PD feedback controller, classically used in control based continuation applications [6], and a phase locked loop controller driving the phase of the reference input signal. The block diagram is depicted in Fig. 1.
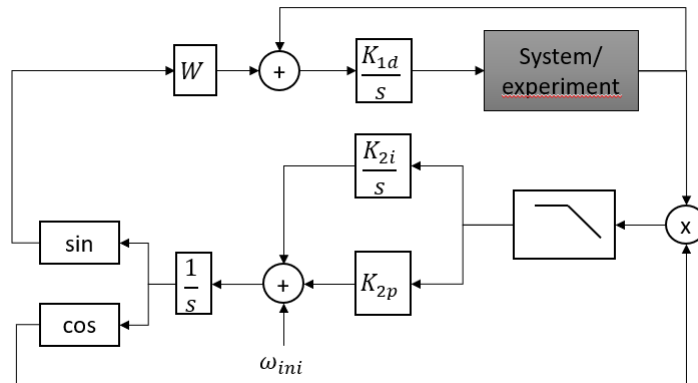

Figure 1: Control strategy

The proposed control scheme has been applied to a Van der Pol oscillator. The governing equations of motion are given by

$$\ddot{x} + x - (\mu - \beta x^2)\dot{x} = K_{1d}(\dot{w} - \dot{x})$$
$$\dot{y}_1 = \omega_c(x\cos\theta - y_1)$$
$$\dot{y}_2 = Ry_1 \tag{1}$$
$$\dot{\theta} = \omega_{ini} + K_{2p}y_1 + K_{2i}y_2$$

where μ and β are the linear and nonlinear coefficients of the Van der Pol oscillator. K_{1d} is the derivative gain (the proportional gain has been omitted for simplicity), ω_c the cutoff frequency of the low pass filter of the phase locked loop, R, K_{2p}, K_{2d} are the gains of the phase locked loop and ω_{ini} the center frequency. $w(t) = W\sin\theta(t)$ is the control target. The uncontrolled Van der Pol oscillator has a Hopf bifurcation at $\mu = 0$, which is subcritical (supercritical) for $\beta < 0$ ($\beta > 0$). Clearly, the controller is non-invasive if $x(t) = w(t)$.
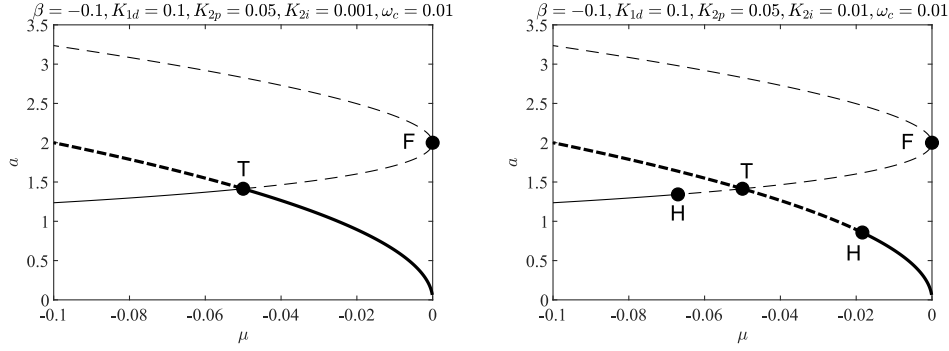

Figure 2: Bifurcation diagram of the controlled VdP

The behavior of the controlled Van der Pol oscillator has been analyzed by using the method of multiple scales. Since the objective is to stabilize the unstable limit cycle, we considered that the control target amplitude W is equal to the amplitude of the uncontrolled limit cycle, i.e. $W = a_0 \equiv 2\sqrt{\mu/\beta}$. Examples of bifurcation diagram are depicted in Fig. 2. Solid (dashed) lines correspond to stable (unstable) solutions. It is shown that in addition to the targeted limit cycle for which the control is non-invasive (represented by thick lines), the coupled system also exhibit control-induced periodic solutions for which the controller is invasive, i.e. $W \neq a_0$ (represented by thin lines). The stability analysis reveal two possible failure scenario of the controller. Either the controller may fail due to the presence of a transcritical bifurcation at $\mu = -K_{1d}/2$ or due to a Hopf bifurcation if $K_{2p}(K_{1d} + 2\omega_c) < 2RK_{2i}$.

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